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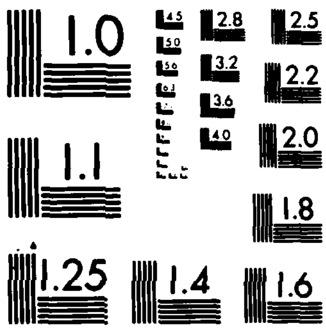

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ON THE PROBABILITY DENSITY FUNCTION OF THE CREST TO TROUGH HEIGHTS OF WAVES AND ON THE PHYSICS OF EXTREME WAVES INCLUDING RESULTS FROM HURRICANE CAMILLE

by

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ABSTRACT

A sequence of 11,000 waves that passed an ODGP platform during Hurricane Camille is studied as 55 successive samples of 200 waves each to determine whether or not the Rayleigh probability density function fits the crest to trough heights defined in the conventional manner and whether or not the highest wave in the sample, which was 72.2 feet from crest to trough, was highly improbable in terms of extreme value theory.

Five reasons why a sample of wave heights can depart from the theoretical Rayleigh distribution are analysed. Two are theoretical; two have to do with sampling problems; and the last is concerned with spectral estimation procedures. It was not possible to isolate each of the five reasons. The results do however show that if the last three are taken into account, the Rayleigh distribution comes close to describing the samples adequately.

The highest waves that occurred were not unusual when tested against extreme value theory.

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INTRODUCTION

There are some indications that the significant wave height in a random sea is not correctly predicted by $4(m_o)^{\frac{1}{2}}$ where m_o (or E_o, or μ_o depending on the author) is the variance of the random waves and the waves are thought of as passing a fixed point as a function of time. Even more, this implies that the crest to trough wave heights do not have a Rayleigh probability density function. In particular, if E_o is the area under the variance spectrum, the pdf

$$f(h) = \frac{h}{4\ E_o} \exp(-h^2/8E_o) \tag{1}$$

and the cumulative density function (cdf)

$$F(h) = 1 - \exp(-h^2/8E_o) \tag{2}$$

do not appear to describe the waves very well.

The short crested Gaussian (and hence linear) seaway has served its purpose well as a first step in departing from the oversimplified models of three of four decades ago, and many of the concepts can be and are still being used for practical applications. Waves, nevertheless, are nonlinear, and over the past decade, or so, there have been many attempts to treat non-linear aspects of waves combined with other attempts to treat linear models without the restrictive assumptions implied by the Rayleigh pdf. A review of much that is known about the probabilistic and statistical properties of waves has been given by Ochi (1982). Substantial advances have been made since the work of Rice (1944), and much of the linear theory still has useful applications.

If equations (1) and (2) are not good enough to describe ocean wave heights with sufficient accuracy, and if in particular

$$H_{1/3} = 4(E_o)^{\frac{1}{2}} \tag{3}$$

is an incorrect equation and the average of the heights of the one third highest waves in an ocean wave time history is not related to the wave spectrum in this simple way, then better models must be found.

The questions that naturally arise if equations (1) to (3) are inadequate are:

1) What probability density functions can be used in place of equation (1) and under what circumstances are they applicable?

2) How can it be proved by means of a sample of crest to trough wave heights from a wave record that a replacement pdf is superior to equation (1) and that it gives better predictions for the wave heights?

3) An extreme value theory based on equation (1) has been developed. If equation (1) does not apply, what are the implications with reference to the prediction of extreme waves?

Historical records are replete with reports , photographs and recordings of extreme waves. Coles (1966) describes them as "freak" waves and refers to them 19 times in a 304 page book with end plates that look remarkably like some figures from Longuet-Higgins and Cokelet (1976). "Freak" troughs are described by Lawrence Draper in an appendix to Cokelet's book. A partial quote from this appendix follows where dashes show parts of the full text have been omitted.

"Reports of freak waves usually concern waves with unexpectedly high crests, as in most of the instances described in previous chapters, but there is just as much chance of an unusually low trough occurring. The reason why they are not often reported must be that a high crest can be seen from a large distance, but a vessel would have to be on the very edge of a deep trough to notice it. Two reports of deep troughs (at least, that is what we believe them to have been) were described in the *Marine Observer* under the heading "The one from nowhere". The following is an extract from the report of Commodore W. S. Byles, R. D., master of the *Edinburgh Castle*, and a similar report of a wartime experience by Commander I. R. Johnston, R. N. (Retired), appearing in that article:" ---

"To further ensure that no untoward incident should occur, I took a knot off her speed and, to close the coast I had, of course, put the swell cosily on the bow instead of driving into it head-on. Under these conditions she was very confortable for three-quarters of an hour or so. The distance from one wave top to the next was about 150 ft. and the ship was pitching and scending about 10-15 degrees to the horizontal. And then it happened. Suddenly, having scended normally, the wave length appeared to be double the normal, about 300 ft., so that when she pitched she charged, as it were, into a hole in the ocean at an

angle of 30 degrees or more, shovelling the next wave on board to a height of 15 or 20 ft. before she could recover, as she was "out of step"."

"Commodore Byles's article was reported in the national Press and brought the following comments from Commander I. R. Johnston, R.N. (Retired):" -----

"We were about 100 miles south-south-west of Durban on our way to Cape Town, steaming fast but quite comfortably into a moderate sea and swell when suddenly we hit the "hole" and went down like a plummet into the next sea which came green over A and B turrets and broke over our open bridge. I was knocked violently off my feet, only to recover and find myself wading around in 2 ft. of water at a height of 60 ft. above normal sea-level."

Buckley (1983) refers to extreme waves as "episodic" waves and has collected a large number of reports on the effects of these waves on ships structures. He also reproduces the time history records of five of the extreme waves recorded during Hurricane Camille.

Questions that arise in connection with extreme (and rare) high waves are:

1) How do they form?

2) Are they a natural, and explainable, consequence of the randomness of the waves?

3) How must mathematical-probabilistic models of a seaway be corrected to account for them?

The problems of a replacement (or replacements) for the Rayleigh pdf and of the prediction and description of extreme waves raise more questions than can be answered completely. However, both presently available experimental data and aspects of newly developed theories (which can be shown to lead to some strange contradictions) point the way to some partial answers and some tentative conclusions.

DATA FROM HURRICANE CAMILLE

Ocean wave time histories obtained by the ODGP program during Hurricane Camille are studied in terms of the series of crest to trough heights so as to determine if they do or do not form an independent random sample and how well the Rayleigh probability density function fits the sample wave height distribution and how well extreme value theory predicts the highest waves in samples 200 consecutive waves.

Hurricane Camille was one of the most thoroughly studied hurricanes in history, and the data on waves from it, as gathered from the Ocean Data Gathering Program (ODGP) of a consortium of oil companies has had extensive study. Examples of research and papers on the waves in this hurricane are given by Ward (1974), Ward, et al. (1978), Cardone, Pierson and Ward (1976), Spring (1978) and Buckley (1983). The study of the probability density function of wave heights by Spring (1978) showed that even the additional freedom to choose a value other than E_O could not provide a better fit and that minor empirical variants of the Rayleigh pdf did not do much better. This present study is a continuation of research completed in March 1982 for DTNSRDC (Pierson and Salfi (1982) (unpublished)). Pertinent results from that study are included. Many of our results are superceded by this present study. This present study parallels much of the difficult and time consuming work done by Spring in the development of computer programs to describe the heights and periods of the waves. Some of the spectral wave properties at ODGP Station 1, as a function of time as Camille approached that offshore oil rig, were also studied by Spring. We provide a brief review of some of the results given by Spring, before proceeding with our own particular analysis. It was possible to obtain data that were in a form very similar to that of Spring, but the data were interpreted in a somewhat different way in terms of the theoretical derivations as given in the following sections of this paper.

One of the first steps in the analysis of Spring, was to break the continuous 22.5 hour time history of the rise and fall of the sea surface at Station 1, into samples of approximately 7.5 minutes in duration. Table 1 of this study is a copy of Table 4 from the work of Spring, in which the 7.5 minute segments were treated as a total of 180 records throughout the duration of the recorded data. Only the first five columns

of the table produced by Spring, are reproduced because the remaining data in his analysis were specific to his particular method of study. Table 1 (of this study) gives the record number, the significant wave height, computed as the average of the one-third highest crest to trough wave heights in the time history, the spectral width parameter, to be defined later, the number of waves, and the significant wave height computed from 4.005 times the square root of the area under the variance spectrum. (Forrestall (1978)).

As can be seen from inspection of this table, the significant wave height computed from the data was generally less than the significant wave height computed from the area under the spectrum. The number of waves decreases from 96 waves in Record Number 1 to 39 waves in Record Number 180, at the peak of the storm measured, or counted, during the 7.5 minute sub-sample. The spectral width parameter is highly erratic, from one record to the next.

A sample of 7.5 minutes duration can yield highly variable estimates of spectra, and although the number of waves during the early part of the total record is fairly high, there do not appear to be enough waves in the 7.5 minute samples toward the end of the series under study to yield an adequate sample.

Table 2 of this paper reproduces Table 5 from Spring in much the same way as the preceeding Table 1 reproduced Table 4. Here the records are fifteen minutes long, and there are half the number of records compared to the preceeding data. Again the significant height computed from this new larger sample, the spectral width parameter and the significant height computed from the area under the spectrum are given. It is noted that Records 1 and 2 of Table 1 contain 192 waves, whereas Record 1 of Table 2 contains 195 waves, or three more waves than Records 1 and 2 of Table 1. These descrepancies have to do with the assumptions made at the ends of each sample record when it was divided into 15 minute versus 7.5 minute samples. For the shorter samples of the waves the ends of the small samples would be omitted if they did not go through a complete cycle. Again it is noted that the significant height computed from the area under the spectrum tends to be larger than the significant height computed from the actual data.

Finally, Table 3, which is Table 6 from Spring shows the results of

samples of 30 minute duration. Record 1 for example, contains 393 waves, as opposed to 387 waves for combined Records 1 and 2 of Table 2. The same thing is happening as the data are pooled in even larger samples. Again the significant wave height computed from actual wave heights tends to be smaller than the height computed from the area under the spectrum.

Figures 1 and 2 are graphs of the significant wave heights as a function of time as plotted from Tables 1. 2 and 3. Figure 1 shows the significant wave height computed from the 7.5 minute samples. Unless indicated by x' , the upper value as a dot is the significant wave height computed from the area under the spectrum, and the lower value, as shown by a dot, is the significant wave height computed from the 7.5 minute sample. There are four records for which the reverse situation occurred, and the significant wave height from the spectrum was lower than the value from "bump counts". These are shown by x's. This graph does not start at the smallest values of the significant wave height. It starts at a somewhat later time than the tabulated data in order to fit it conveniently to the graph paper that was used. The start is Record Number 31.

Figure 1 also shows some of the effects of sampling variability from one 7.5 minute sample to another; the fluctuations between Record 110 and 130, Record 121 in particular, and the sharp jump from Record 135 to 136, are indicative of an effect of small sample size. As the waves get higher, and the number of waves in the 7.5 minute samples decrease, the irregularities become very pronounced. For example, Record 180 indicates a significant wave height of 50.5 feet, for one method of analysis and 52.5 feet for the other method of analysis. The one 7.5 minutes earlier indicates significant wave heights of 33.5 and 35.5 for the same conditions for the same record length. The change is rather dramatic.

The data for the 15 minute samples and the 30 minute samples are combined in Figure 2. The fifteen minute samples are shown below and the thirty minute samples are shown above, with a displaced coordinate system. In both cases the significant wave height approaches a value slightly over forty feet.

Figure 3 is a graph of the spectral width parameter for successive 15 minute samples. It varies very erratically. A five point running mean of of the spectral width parameter is shown with the scale on the right.

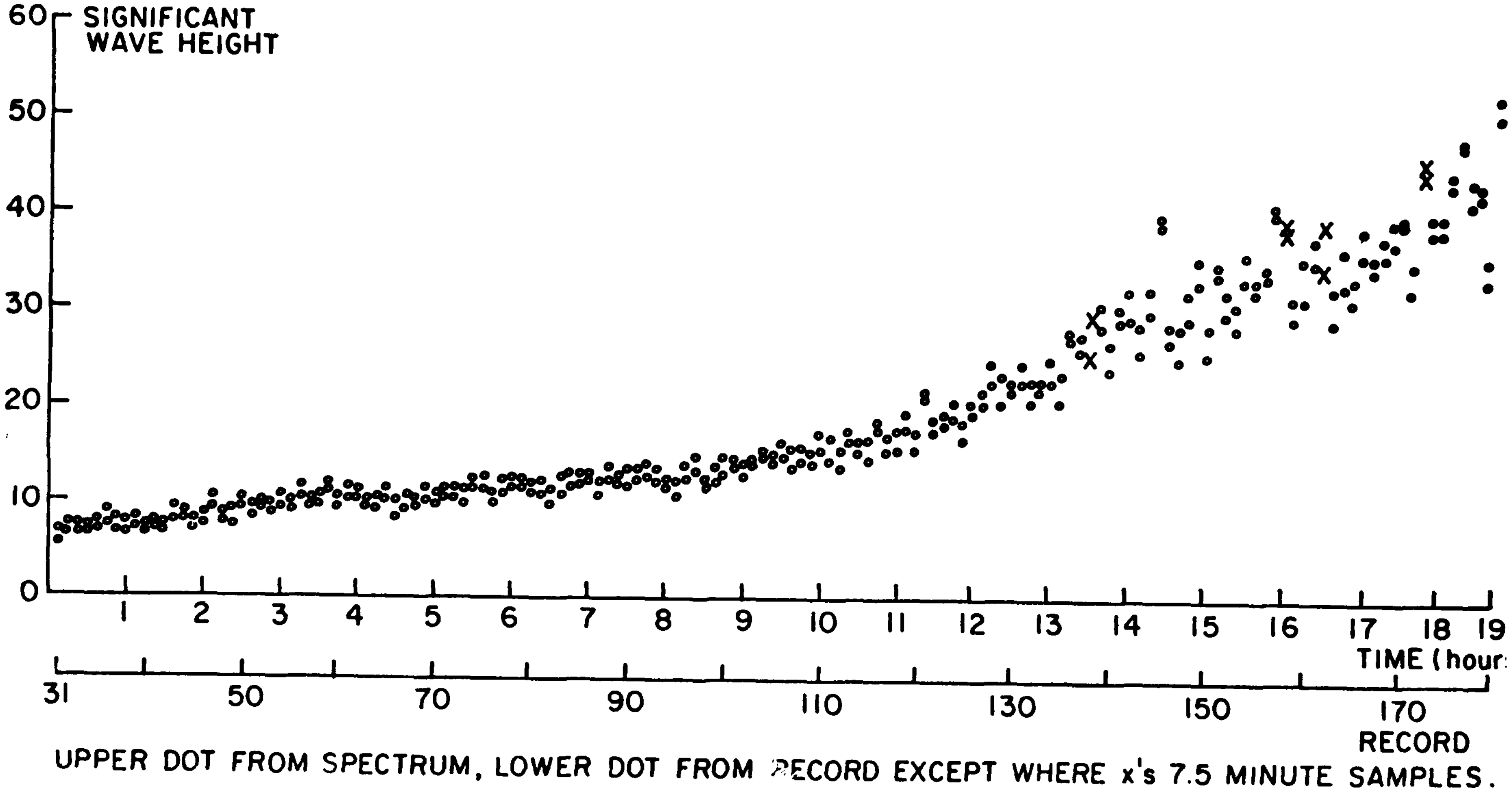


FIGURE 1 Significant Wave Height Versus Time for 7.5 Minute Samples.

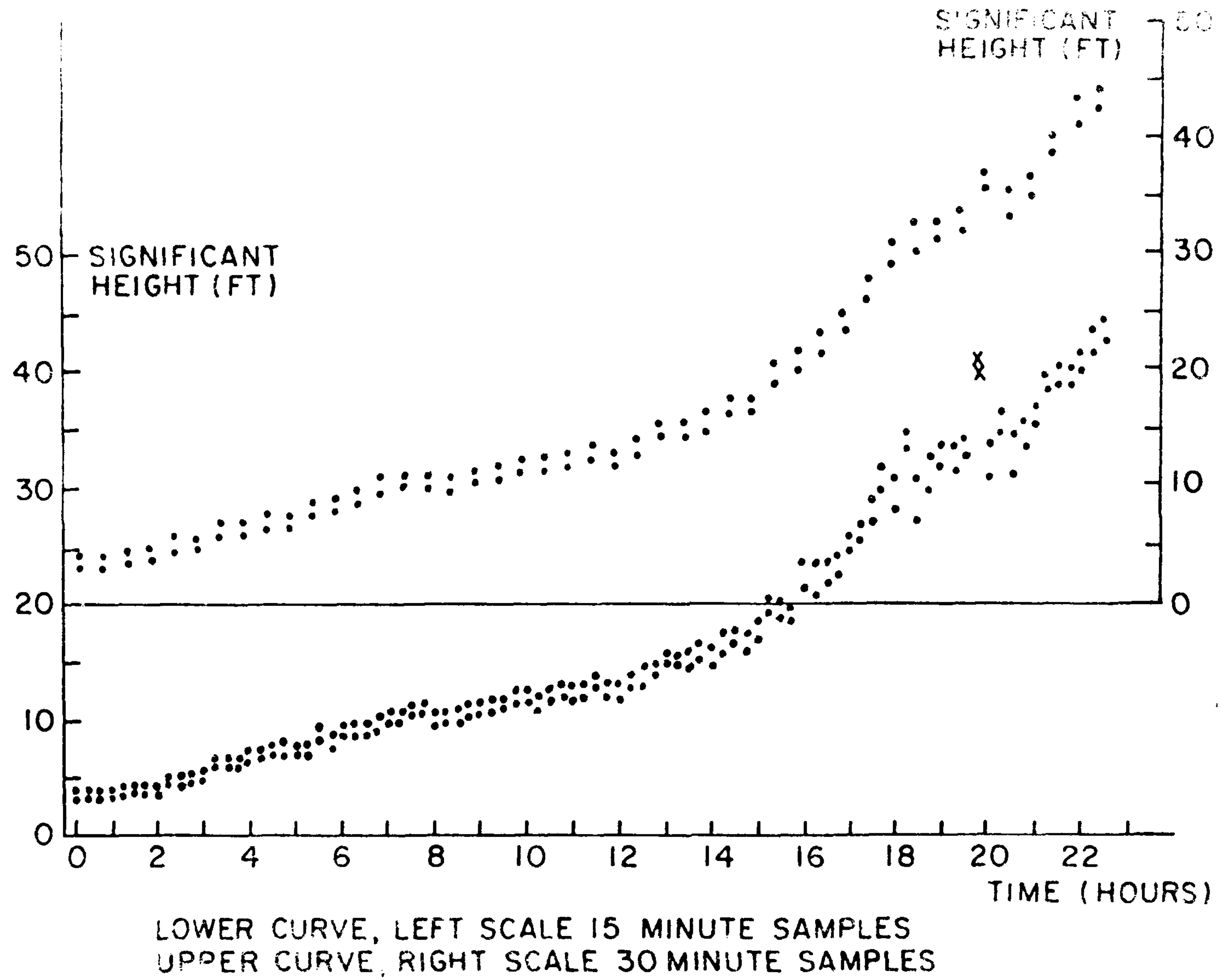


FIGURE 2. Significant Wave Height Versus Time.

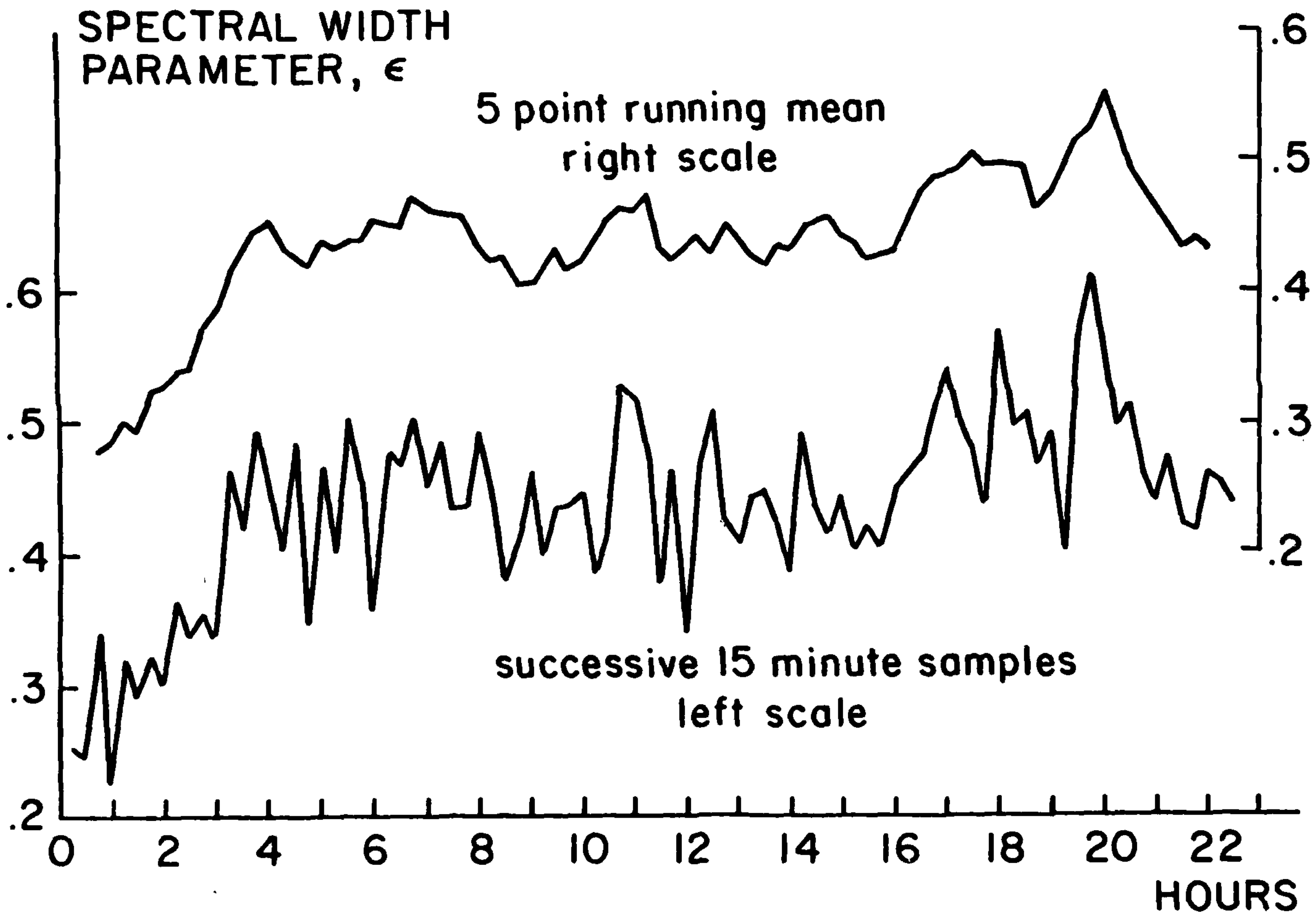


FIGURE 3   Spectral Width Parameter as a Function of Time for 15 Minute Samples.

TABLE 1A Seven and One Half Minute Samples from Hurricane Camille; H_s Significant Height from Data; SWP, Spectral Width Parameter: RH_s, Significant Height from Spectrum. Records 1 to 35.

REC	H_S	SWP	#WAVES	RH_S
1	3.29	.224308	96	3.67
2	3.23	.315101	96	3.70
3	3.19	.305556	90	3.60
4	3.12	.265306	96	3.72
5	3.10	.305556	90	3.62
6	3.22	.374463	87	3.22
7	3.02	.195039	93	3.71
8	3.51	.305556	90	3.90
9	3.77	.372019	84	4.21
10	3.71	.318239	90	4.27
11	3.81	.305556	90	4.36
12	3.96	.330579	90	4.33
13	3.98	.338894	87	4.66
14	3.76	.362932	87	4.30
15	3.78	.286549	87	4.60
16	3.72	.362932	87	4.52
17	4.25	.383702	84	4.73
18	4.42	.393399	81	4.96
19	4.50	.308303	84	5.22
20	4.37	.415225	78	4.83
21	4.37	.321799	84	4.97
22	4.66	.437500	78	5.37
23	4.60	.334904	84	5.18
24	5.05	.426079	75	5.59
25	6.23	.504269	69	7.00
26	5.55	.426079	75	6.54
27	6.22	.449532	69	7.24
28	5.47	.426079	75	6.45
29	5.71	.402168	75	6.79
30	5.81	.576273	69	6.52
31	5.85	.403588	78	6.85
32	6.71	.493995	69	7.46
33	6.69	.335421	75	7.47
34	6.48	.993995	69	7.02
35	6.91	.483398	69	7.76

TABLE 1B Continuation of Table 1A, Records 36 to 70.

REC	H_S	SWP	#WAVES	RH_S
36	7.18	.483398	69	8.72
37	6.78	.425596	72	8.08
38	6.77	.305556	75	7.87
39	7.29	.550815	63	8.15
40	6.57	.414307	75	7.55
41	7.22	.389648	75	7.99
42	6.70	.402168	75	7.54
43	7.99	.565270	60	9.23
44	8.10	.483398	69	8.94
45	7.13	.472465	69	7.95
46	7.86	.449038	72	8.63
47	9.23	.463359	63	10.34
48	7.74	.248889	78	8.87
49	7.82	.426079	75	9.01
50	9.39	.566330	54	10.28
51	8.15	.531073	63	9.22
52	9.26	.437500	63	9.96
53	8.89	.570749	57	9.65
54	9.50	.473977	66	10.76
55	9.15	.531073	63	9.75
56	10.30	.411033	66	11.67
57	9.31	.564408	66	10.03
58	9.71	.370987	69	10.84
59	11.19	.528379	57	11.74
60	9.61	.349636	75	10.54
61	10.31	.382653	66	11.30
62	10.26	.987474	63	11.09
63	9.30	.496358	66	10.28
64	9.16	.541103	63	10.81
65	10.17	.423864	63	11.17
66	8.62	.504269	69	10.16
67	9.09	.363400	75	10.71
68	9.42	.376731	75	10.70
69	10.06	.385201	69	11.31
70	9.93	.924495	66	10.91

TABLE 1C Continuation of Table 1A, Records 71 to 105.

REC	H_S	SWP	#WAVES	RH_S
71	10.44	.545512	60	11.08
72	10.62	.425069	69	11.60
73	9.80	.424495	66	11.25
74	11.34	.367689	66	12.35
75	11.23	.437500	63	12.47
76	10.01	.425596	72	11.02
77	10.98	.411033	66	12.26
78	11.46	.450657	63	12.45
79	11.49	.423169	60	12.48
80	10.91	.501730	60	11.85
81	10.90	.409726	63	12.11
82	9.74	.397093	66	11.47
83	10.89	.274348	69	12.40
84	11.74	.510000	63	12.99
85	11.95	.524376	60	12.94
86	12.05	.570749	57	13.03
87	10.92	.464604	60	12.07
88	12.28	.555556	60	13.53
89	11.84	.463359	63	12.89
90	11.66	.487474	63	13.50
91	12.38	.376731	60	13.20
92	12.81	.423169	60	14.05
93	12.01	.450657	63	13.38
94	11.63	.535124	60	12.46
95	10.48	.274348	69	11.99
96	12.53	.450657	63	13.95
97	13.06	.539541	57	14.94
98	11.50	.437500	63	12.37
99	11.99	.539541	57	13.58
100	12.93	.528379	57	14.90
101	13.91	.464404	60	14.41
102	12.90	.379844	63	14.04
103	14.34	.364044	63	14.45
104	14.80	.495152	54	15.76
105	14.05	.463359	63	14.59

TABLE 1D, Continuation of Table 1A, Records 106 to 140.

REC	H_S	SWP	#WAVES	RH_S
106	14.56	.475624	63	16.26
107	13.47	.504801	57	15.70
108	14.16	.437500	60	15.81
109	14.12	.463359	60	15.32
110	15.79	.373264	57	17.43
111	14.20	.492344	57	16.83
112	13.70	.390317	57	15.31
113	16.26	.465976	57	17.54
114	15.07	.504801	57	16.56
115	14.47	.422400	57	16.60
116	17.79	.511916	51	18.68
117	15.53	.525018	51	16.87
118	15.56	.5081809	54	17.43
119	17.91	.511916	51	19.14
120	15.68	.421543	54	17.14
121	21.10	.419501	48	21.89
122	17.24	.453686	51	18.90
123	18.13	.369377	54	19.43
124	19.00	.525018	51	20.84
125	16.81	.297362	57	18.57
126	19.63	.549688	51	20.75
127	20.15	.525018	51	21.63
128	22.90	.555556	48	24.81
129	20.42	.437500	51	23.54
130	21.56	.537600	51	22.56
131	22.45	.453686	51	24.25
132	20.44	.453686	41	22.77
133	21.81	.542948	48	22.69
134	22.77	.529796	48	25.00
135	20.91	.574669	45	23.45
136	27.17	.562067	45	27.92
137	26.00	.520710	45	27.44
138	29.43	.542948	48	25.82
139	28.24	.520710	45	30.84
140	24.01	.555556	48	26.91

TABLE 1E, Continuation of Table 1A, Records 141 to 175.

REC	H_S	SWP	#WAVES	RH_S
141	28.99	.525934	42	30.57
142	29.41	.419501	48	32.11
143	25.91	.574669	45	28.84
144	29.94	.618512	42	32.19
145	39.07	.493249	42	39.81
146	26.93	.562067	45	28.33
147	24.98	.473205	45	28.3
148	29.11	.535124	45	32.05
149	33.06	.455791	45	35.77
150	25.47	.469184	51	28.73
151	34.12	.547860	39	34.84
152	29.94	.338121	48	32.01
153	28.32	.520710	45	30.72
154	33.85	.221453	45	36.16
155	32.26	.541103	42	33.18
156	33.60	.591239	39	34.93
157	40.21	.664820	33	40.88
158	39.16	.618512	42	38.84
159	29.25	.607039	42	31.32
160	31.26	.555556	42	35.31
161	35.06	.486745	48	37.32
162	39.18	.562067	45	34.90
163	29.13	.582485	42	32.06
164	32.99	.505615	45	36.27
165	31.21	.535124	45	33.03
166	35.81	.380812	48	38.33
167	34.05	.505615	45	35.95
168	36.06	.437500	45	37.68
169	37.57	.473205	45	39.57
170	39.42	.541103	42	39.55
171	32.35	.525934	42	34.94
172	45.10	.478395	39	44.61
173	38.69	.398038	45	40.11
174	38.37	.437500	45	40.01
175	43.07	.555556	42	44.20

TABLE 1F, Continuation of Table 1A, Records 176 to 180.

REC	H_S	SWP	#WAVES	RH_S
176	47.38	.437500	45	47.94
177	41.37	.505615	45	43.63
178	42.43	.473205	45	42.99
179	33.23	.384379	51	35.41
180	50.24	.478395	39	52.29

TABLE 2A, Fifteen Minute Samples from Hurricane Camille; see Table 1A, Records 1 to 35.

REC	H_S	SWP	#WAVES	RH_S
1	3.26	.255521	195	3.69
2	3.16	.245224	192	3.66
3	3.18	.340775	177	3.65
4	3.30	.227236	189	3.80
5	3.74	.322250	177	4.24
6	3.88	.295324	183	4.34
7	3.88	.328511	177	4.48
8	3.78	.302933	177	4.56
9	4.35	.366052	168	4.85
10	4.46	.339343	165	5.03
11	4.52	.358446	165	5.17
12	4.78	.339343	165	5.39
13	5.94	.465691	144	6.77
14	5.88	.419877	147	6.85
15	5.82	.496809	144	6.66
16	6.31	.454332	147	7.16
17	6.60	.401413	147	7.25
18	7.13	.488738	138	7.99
19	6.77	.342586	150	7.97
20	6.98	.466268	141	7.85
21	6.92	.395957	150	7.77
22	8.15	.507017	132	9.09
23	7.44	.460693	141	8.30
24	8.53	.358181	141	9.63
25	8.67	.473977	132	9.67
26	8.72	.468831	120	9.60
27	9.11	.504510	126	10.22
28	9.70	.450069	132	10.75
29	9.56	.484354	135	10.44
30	10.41	.437500	132	11.16
31	10.29	.437500	129	11.19
32	9.21	.496358	132	10.55
33	9.47	.443698	135	10.67
34	9.40	.376731	150	10.70
35	10.02	.358201	138	11.11

TABLE 2B, Continuation of Table 2A, Records 36 to 70.

REC	H_S	SWP	#WAVES	RH_S
36	10.48	.462222	132	11.34
37	10.53	.397093	132	11.81
38	10.79	.437500	135	11.77
39	11.22	.437500	129	12.36
40	11.29	.444298	123	12.17
41	10.44	.382653	132	11.79
42	11.42	.411033	132	12.70
43	11.93	.529796	120	12.99
44	11.65	.518861	120	12.82
45	11.78	.475624	126	13.20
46	12.53	.378328	123	13.63
47	11.74	.469545	126	12.93
48	11.58	.338620	135	13.00
49	12.60	.470292	123	13.71
50	12.57	.509161	117	14.26
51	13.41	.423525	123	14.79
52	14.80	.408284	120	15.61
53	14.27	.443984	129	15.45
54	14.07	.444466	120	15.75
55	14.81	.423169	120	16.41
56	14.37	.384852	120	16.09
57	15.64	.492344	114	17.06
58	16.49	.437500	111	17.67
59	15.57	.516469	105	17.15
60	16.77	.445232	108	18.17
61	19.09	.402893	102	20.45
62	18.62	.421543	108	20.15
63	18.38	.405816	111	19.69
64	21.58	.450205	99	23.27
65	20.90	.460854	105	23.06
66	21.37	.421078	105	23.52
67	22.17	.513936	99	23.87
68	24.51	.539187	93	25.79
69	25.10	.501730	96	26.65
70	26.59	.477809	99	28.94

TABLE 2C, Continuation of Table 2A Records 71 to 90.

REC	H_S	SWP	#WAVES	RH_S
71	29.57	.437500	93	31.35
72	27.96	.568437	90	30.57
73	33.07	.497799	90	34.55
74	26.91	.505615	90	30.24
75	29.67	.462968	96	32.45
76	31.89	.447074	87	33.45
77	31.11	.398038	90	33.55
78	32.65	.566330	81	34.07
79	40.62	.610624	78	39.87
80	30.86	.552130	87	33.34
81	34.41	.494321	96	36.13
82	30.99	.513250	90	34.23
83	33.52	.463761	93	35.78
84	35.29	.437500	93	36.83
85	38.26	.473205	90	39.56
86	38.70	.427675	87	40.07
87	38.87	.418271	90	40.06
88	44.94	.464604	90	46.11
89	41.56	.455215	93	43.32
90	42.68	.437500	90	44.66

TABLE 3A, Thirty Minute Samples from Hurricane Camille, See Table 1A. Records 1 to 35.

REC	H_S	SWP	#WAVES	RH_S
1	3.19	.230464	393	3.69
2	3.24	.273765	369	3.73
3	3.79	.285526	366	4.30
4	3.83	.319087	354	4.52
5	4.38	.356139	333	4.94
6	4.68	.349000	330	5.28
7	5.91	.446099	291	6.81
8	6.02	.465137	294	6.91
9	6.85	.425720	291	7.63
10	6.88	.398250	294	7.19
11	7.53	.440449	285	8.45
12	8.04	.400217	285	8.99
13	8.69	.462222	264	9.63
14	9.42	.480634	258	10.99
15	10.03	.452805	270	10.80
16	9.82	.459222	264	10.88
17	9.43	.384769	291	10.69
18	10.25	.428006	270	11.22
19	10.66	.404898	270	11.79
20	11.25	.430824	255	12.26
21	11.06	.397093	264	12.25
22	11.86	.515201	243	12.90
23	12.32	.416860	252	13.42
24	11.65	.393531	264	12.97
25	12.57	.476961	243	13.99
26	14.11	.401566	246	15.21
27	14.27	.4442.6	249	15.60
28	14.68	.404473	240	16.25
29	16.07	.437500	231	17.37
30	16.22	.485665	213	17.67
31	18.83	.395692	213	20.30
32	20.01	.464236	213	21.56
33	21.14	.441496	210	23.29
34	23.34	.526419	192	24.85
35	25.96	.493513	195	27.82

TABLE 3B, Continuation of Table 3A. Records 36 to 45.

REC	H_S	SWP	#WAVES	RH_S
36	29.08	.508389	183	30.96
37	30.11	.484983	183	32.47
38	31.08	.442009	186	32.95
39	31.95	.474375	174	33.81
40	35.53	.584579	165	36.75
41	32.75	.487474	189	35.19
42	34.66	.433009	189	36.31
43	38.45	.451533	177	39.81
44	41.95	.442158	180	43.19
45	42.29	.451081	183	44.00

A major change in the way that the data were processed in this study was to regroup the successive waves into non-overlapping sets of 200 waves each no matter how long in time the sample lasted. Reasons for doing this will become evident as our study progresses.

One result of this re-arrangement is shown in Fig. 4. The horizontal axis shows time in minutes (and hours) during the passage of the waves. The left axes show wave height in feet and meters and apply to the circled dots. The right axis shows the average of the "periods" of the waves in the 200 wave sample which is shown by a "square" on the figure. The circled dots show the highest wave in each 200 wave sample. The various numbers by each point are explained on the figure.

Similar data are also shown in tabular form as in Table 4. There were 55 two hundred wave samples as shown by the first column. The time it took the 200 waves to pass in minutes is shown in the second column from which the average period can be found. The third column shows the height of the highest wave and the last column shows the period of that highest wave. The highest wave in the entire sequence is in record 48 and is 72.23 feet from crest to trough. The overall trend for the highest wave is upward, but the variability from one record to the next is considerably larger than the variability shown in Figure 3.

These 55 samples will be studied in two different ways after theoretical results of various authors are described.

Just prior to providing a summary, some tentative conclusions, and a discussion of the implications of our results for wave forecasting, wave modeling, and the study of the motions of ships in waves, the work of Spring (1978) will be reinterpreted in view of both the theoretical and practical problems that arise in the study of ocean wave time histories.

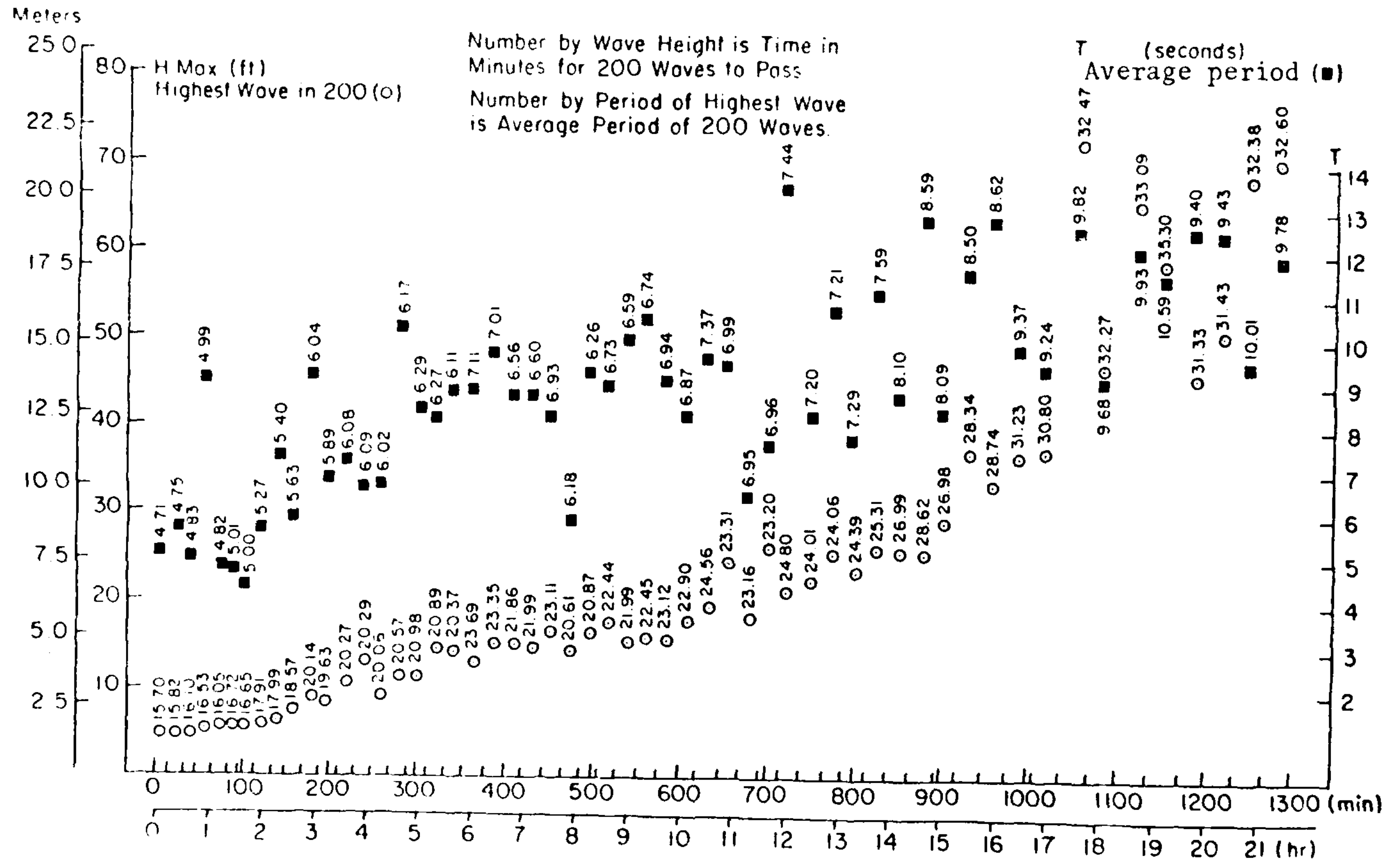


FIG 4 Maximum Wave in Each Sample and Average Period of the 200 Waves.

RECORD	LEN(MIN)	HMAX(FT)	TMAX(SEC)
1	15.73	4.81	5.16
2	15.82	4.48	5.60
3	16.10	4.61	4.93
4	16.63	5.33	8.98
5	16.05	5.69	4.78
6	16.72	5.84	4.63
7	16.65	5.89	4.32
8	17.91	6.07	5.73
9	17.99	6.44	7.31
10	19.57	7.67	5.93
11	20.14	9.03	9.09
12	19.63	8.21	6.77
13	20.27	10.97	7.16
14	20.29	13.33	6.58
15	20.26	9.19	6.62
16	20.57	11.81	10.20
17	20.98	11.58	8.37
18	20.89	14.70	8.19
19	20.37	14.65	8.76
20	23.69	13.08	8.76
21	23.35	15.34	9.62
22	21.86	15.29	8.61
23	21.99	14.90	8.65
24	23.11	16.96	8.16
25	20.61	14.39	5.80
26	20.87	16.65	9.22
27	22.44	17.43	8.37
28	21.99	15.60	9.90
29	22.45	16.09	10.41
30	23.12	15.93	8.96
31	22.90	17.97	8.26
32	24.56	19.43	9.55
33	23.31	24.35	9.37
34	23.16	18.35	6.35
35	23.20	26.15	7.59
36	24.80	21.08	13.42
37	24.01	22.39	8.25
38	24.06	25.69	10.61
39	24.39	23.47	7.69
40	25.31	26.02	11.03
41	26.99	26.13	8.62
42	29.62	25.56	12.67
43	26.98	29.06	8.28
44	28.34	37.09	11.44
45	28.74	33.62	12.70
46	31.23	36.76	9.72
47	30.80	37.19	9.33
48	32.47	72.23	12.45
49	32.27	46.46	9.09
50	33.49	65.41	12.14
51	35.30	59.94	11.40
52	31.33	45.77	12.52
53	31.43	50.66	12.41
54	33.38	68.47	9.46
55	32.60	70.35	11.84

TABLE 4 Durations in Minutes, Maximum Wave Height in 200 Wave Sample, and Period of the Maximum Wave for 55 Samples Hurricane Camille.

STUDIES OF RANDOM WAVES AND OF RANDOM WAVE SPECTRA

Introduction. Although, this investigation is concerned with the description of the crest to trough wave height pdf, whether or not equations (1), (2) and (3) describe the wave heights, and if not how can a better model be found, there is a connection to the problems associated with spectral estimation, because the alternative models to be described involve higher moments of the spectra in addition to m_o (or E_o). If the spectra are not well known (and, for some applications, well predicted), the use of more sophisticated models may not be warranted. For this reason, some recent results in the area of spectral estimation are reviewed in this section.

Review of Concepts. Waves that have been generated on the ocean by the wind are random in both space and time. Nevertheless, their behavior must satisfy the hydrodynamic equations. The model of waves as a short crested stationary (probabilistically) Gaussian process made up of the superposition of sinusoids is an approximation to reality since waves are in fact non-stationary (i.e. evolutive) and non-Gaussian. If they were not, there would be no need to measure them or predict them.

The basic fundamental concept, which Tukey (1949) called "Power Spectral Analysis" of a sample from a Gaussian time history, as an extention of the earlier seminal work of Rice (1944), and which Blackman and Tukey (1958) applified upon in a second key paper is that time (or space) series of data, which are highly correlated in time and, or, space (hence the hydrodynamics for waves), can be *sampled* and analysed to obtain an estimate of the spectrum that resolves the variance of the random process into frequency and/or wavenumber bands. In the spectral domain, the complicated time and space interdependence in physical space (i.e. x, y, z, t) becomes totally uncorrelated in frequency, or wave number space, for spectra and understandable in terms of cross spectra for physical or hydrodynamical effects (as in bow submergence and slamming).

This basic concept had to be interpreted carefully in their original work, but FFT analysis methods (Cooley and Tukey (1965)) simplified the

problem clean up. The fundamental concept of spectral estimation as a quotation from Blackman and Tukey (1949) is that "Clearly, for any specific value of T_n (on number of pieces of record) we can increase frequency resolution (or stability) only by sacrificing stability (or frequency resolution)" (pg. 93). They also state: "The only major difficulty in their practical application (i.e. spectral estimation) is the extensiveness of the data required in highly precise estimates. Their requirement is an inherent irrevocable characteristic of such random processes". (pg. 2)

Some doubts were expressed about this fundamental concept at a recent conference (Baer (1980)). Moreover, a recent special issue of *Proceedings of the IEEE*, with an historical perspective by Robinson (1982) has papers that describe all sorts of ways to try to avoid the problem of sampling variability.

It is, and always will be, unavoidable!

In the time and space domain, which is the real world, as opposed to the theoretical world of frequencies and wave numbers, because of the temporal and spatial dependence of the data, analysis of the probability structure of the waves becomes ever so much more complicated. Ochi (1982) has reviewed the present state of the subject, with references through 1981, including aspects of extreme value theory that are (1) theoretically based on the Rayleigh pdf, (2) on approximate methods and (3) based on a sample of extreme waves as measured.

Results of Donelan and Pierson. Although spectral analysis has been used for quite a few years, there are ways to misinterpret, and to make incorrect inferences from, spectral estimates. Donelan and Pierson (1983) made an experimental check of the sampling variability of estimates of spectra of wind generated gravity waves under both laboratory (C.C.I.W.) and field conditions (at Lake Ontario). The theory checked extremely well. Nonlinear effects are an add-on correction as far as the basic properties of estimated variance spectra are concerned.

The variability of the sixteen spectra estimated from sixteen time histories measured in the wind wave flume at a constant fetch for a constant wind highlights how variable spectral estimates can be and that neither the significant wave height calculated from equation (3) nor the location of the frequency where the sample spectrum has a peak is well known. The quotations

from Blackman and Tukey (1949) provide a constraint that is unavoidable.

The data processing methods that were used to determine the JONSWAP five parameter family of spectra are an example of the difficulties and contradictions that can arise in interpreting spectra as discussed by Pierson (1977) and by Donelan and Pierson (1983), the latter in a rather restrained way (almost by implication).

Implications for JONSWAP. The study by Günther (1981) is most revealing in that it introduces a fundamental dilemma in the parametric method for forecasting waves. Three spectra with the five JONSWAP spectral parameters chosen to represent a range of conditions were used and normalized to a dimensionless form by $f_m/\Delta f$ so as to select a spectral resolution. The expected value of the normalized variance in each spectral band was thus known. Given the expected value for each spectral band, randomly selected values from a Chi Square distribution with 30 degrees of freedom were used to generate a succession of sample spectra.

As shown by Pierson and Donelan (1983), if the entire set of randomly generated sample spectra had been averaged frequency band by frequency band, the result would have been very close to the input spectra. One hundred samples, if averaged, would have given a spectrum with 3000 degrees of freedom within $\pm$ 4% of the input spectrum at the 90% confidence level.

This was not done, however, each sample spectrum was separately analysed to recover the JONSWAP parameters. The recovered parameters were averaged and their standard deviations were found. The output spectra based on the five averaged parameters were not the same as the input spectra. (See Günther (1981) and Donelan and Pierson (1983)). This result exposes a fallacy in the definition and parameterization of the JONSWAP spectra as claimed by Pierson (1977). It also produces a dilemma.

Suppose for the first choice of the dilemma that a place could be found somewhere on the ocean with constant offshore winds and no effect of depth, that the JONSWAP parametric model (Hasselmann, et al. (1976)) is used to predict the waves as a function of fetch, and that wave records of standard duration are obtained from which the spectra are estimated. (Spectra are not measured-some authors to the contrary.) For each of the spectral estimates, the values of f_m at the same fetch will vary from one spectrum to another

and the value of γ (the peak enhancement parameter) will scatter considerably once f_m is selected. The predicted JONSWAP spectra will not be verified by the individual spectral estimates. Under the assumed conditions, records, say ten or twenty times longer can be obtained. If these are fitted by the JONSWAP method, the values of γ will shrink to much lower values and the values of f_m will tend toward a much more stable value. The predicted JONSWAP spectra will not agree with the spectral estimates, especially in shape.

For the second choice of the dilemma, use all of the above, plus the assumption that the waves that are generated actually do have the JONSWAP spectra predicted by the parametric model. If the sample spectra are used to verify the parametric model predictions, the values of γ will be too high, and individual values of f_m will still scatter in an unpredictable way as shown by Günther's simulation. Under these assumptions, spectra estimated from wave records ten or twenty times longer than usual will be closer to the predicted spectral shapes.

To be on the horns of a dilemma is to be in a position where the choice of either one of two alternatives is equally unfavorable. One choice is to claim that the JONSWAP parameters are a valid way to describe fetch, or duration, limited wave spectra. The parameters will be a function of the length of the "stationary" time history, and sample spectral estimates will not verify. The other choice is to claim that the parametric method predicts the JONSWAP spectral shapes correctly. Were this the correct situation, the spectral peaks of the estimated spectra would all be higher than those of the predicted spectra and the forecast would not verify. Only much longer records would verify the method.

The reader will, hopefully, be able to deduce from the above discussion that the first alternative, which is a result of the data analysis fallacy, corresponds to the actual situation. The spectra predicted by the parametric model are not predictable spectra.

There are numerous implications of the above results, with reference to overshoot phenomenon and the verification of nonlinear wave-wave interaction theories. There is the strong possibility that, even if non-linear wave interactions are possible, their effect has been greatly overestimated when found from estimated wave spectra.

<u>The Surface Contour Radar</u>. The surface contour radar (SCR) has been used by Walsh, et al. (1983) to measure waves as a function of x, y (at rapidly varying t) for a swath beneath an aircraft flight line. Stereo photographs give spectra with an ambiguity of the form, $S(k_x, k_y) = S(-k_x, -k_y)$, but a way has been found to eliminate the false part of the spectrum based on an effect produced by the moving aircraft. For linear waves the spectra can be transformed to the form $S(\omega, \theta)$. Advantages of the SCR are high wave number resolution in one direction and a large number of degrees of freedom for an estimate of $S(\omega,\theta)\ \Delta\omega\Delta\theta$ for typical resolutions.

A study was made of the growth of fetch limited waves for offshore winds of 15 m/s and 22 m/s measured at 400 m. One fetch was about 200 km for a flight exactly downwind that began about 27 km northward along the coast from Cape May New Jersey. The other flight was offshore from the coast of Maryland about 100 km south of Cape Henlopen.

Delaware Bay produced some most interesting effects. The distance from Cape May to Cape Henlopen is about 18 km, and the fetch over Delaware Bay is perhaps twice this distance. The line connecting the two capes can be idealized to a wave generator injecting waves with a spectrum of the form, $S(\omega,\theta)$. The angular spread for a particular frequency can be described in many ways with perhaps the normalized form $K(\cos(\theta/2))$ to the power, $P(\omega)$, being the simplest.

The vector wave number spectrum was estimated (in a statistical sense) at various places along the fetch as the plane flew downwind off the New Jersey Coast. The growth of the waves that would be expected in the absence of Delaware Bay is found in the spectrum. A distinct second spectral peak is also found in each spectrum that can be traced back to the line joining the two capes. The waves emanating from the bay continue to grow, even though moving at an angle to the wind. A modification of spectral growth theories so as to include wind direction and so as to describe a directional wave spectrum would account for these observations since zero fetch would be somewhere inside the bay and the rate of exponential growth for a particular frequency would begin at a distance related to this zero fetch baseline instead of the New Jersey Coastline.

Whether or not the presence of waves from Delaware Bay inhibited the

growth of the waves farther north generated by the offshore wind was also studied. The results were not conclusive, but there was not any convincing data to show any inhibitory effects.

Second Order Effects on Wave Spectra. The work of Masuda, et al. (1979), Kuo, et al. (1979a, 1979b) and Mitsuyasu, et al. (1979) found that (1) the main part of the wave spectrum could be treated as free waves with minor corrections to their phase speeds as a result of nonlinear interactions, (2) that wind wave flumes require consideration of a wind drift current that is relatively more important in the laboratory than in the ocean and that (3) the only nonlinear effects on the spectrum were bound harmonics at frequencies past 1.6 times the spectral peak.

This study was an extention of the work of Tick (1959), who solved the problem for long crested deep water waves with a free surface described by, say, $\eta(x,t)$. Their extension was for a free surface given by $\eta(x,y,t)$, and assumed directional spreads were used. The results were, as would be expected, an improvement if compared to Tick because the waves were short crested.

For ocean wind waves, the correction to the variance spectrum was well past the spectral peak. The correction was more than two orders of magnitude down from the peak of the linear part of the spectrum. Other corrections were more than three orders of magnitude down. Phase speed changes were minimal increases as a function of frequency. The bound harmonics at second order traveled at twice the speed of the fundamental.

Variance spectra are not particularly revealing in the study of non-linear random waves. There is no reason to believe that the correction to the phase speed is simply the same constant everywhere for a given frequency. There are certain assumptions in these results concerned with triple products of various terms that need further checking by other means. They may not be correct.

Generation of Monochromatic Waves by Wind. Mitsuyasu and Honda (1982) have shown spectra of wind waves plus monochromatic waves in which the spikes for the monochromatic wave were three orders of magnitude above the wind wave spectra. It was possible to compute the exponential growth rate of the monochromatic wave as a function of fetch in the presence of wind generated waves.

With the wind wave spectra almost completely eliminated by the addition of a surfactant, it was also shown that monochromatic waves grew at essentially the same rate after a correction for the change in z_o and u_* was made as a result of the surfactant.

One can possibly infer that a spectral component in a wind sea grows independently of the other components and does not depend on nonlinear interactions. The results of Mitsuyasu and Honda (1982) do not fully settle the question since the highest frequency used was somewhat lower than the frequencies in the wind wave spectra.

Nonlinear results as applied to the study of spectra do not seem to be producing dramatic results in these examples. The application of nonlinear theories to the study of waves in terms of wave height distributions and other wave statistics has much still to be done.

NEW NONLINEAR WAVE THEORIES

Introduction. Many new departures on nonlinear wave theories have been initiated over the past few years. Their connection to random waves on the sea surface is not yet clear. There may never be any connection for some of the results that have been obtained. Before plunging into a summary and critique of these results, it appears to be worthwhile to look at Stokes' waves at third order in a somewhat different way.

The free surface, potential function, and phase speed for such a wave, if it exists at all, are given by

$$\eta(x,t) = a\cos\left\{k(x - Ct)\right\} + \frac{a^2 k}{2}\cos\left\{2k(x - Ct)\right\} + \frac{3}{8}a^3 k^2 \cos\left\{3k(x - Ct)\right\} \quad (4)$$

$$\Phi(x,z,t) = -a\,C_o\left(1 - \frac{a^2 k^2}{8}\right) e^{kz} \sin\left\{k(x-Ct)\right\} \quad (5)$$

$$C = \left(\frac{g}{k}\right)^{1/2}\left(1 + \frac{a^2 k^2}{2}\right) \quad (6)$$

The term $(a^2 k^2)/2$ in equation (6) only arises when the third order terms in equations (4) and (5) are found. The correction to the phase speed only arises at third order. Since $kC = \omega$, (6) can be changed to (7).

$$\omega = (gk)^{1/2}\left\{1 + (a^2 k^2)/2\right\} \quad (7)$$

If the frequency of the wave cannot change, then the amplitude of the wave is determined by the wave number as in (8),

$$a = \frac{1}{k}\left\{2\left(\frac{\omega}{(gk)^{1/2}} - 1\right)\right\}^{1/2} \quad (8)$$

For a constant frequency of $2\pi/10$, or a period of 10 seconds, Figure 5 shows the wave profile as a function of time at a fixed point along with the values of a, k, L, the crest height and the trough depression.

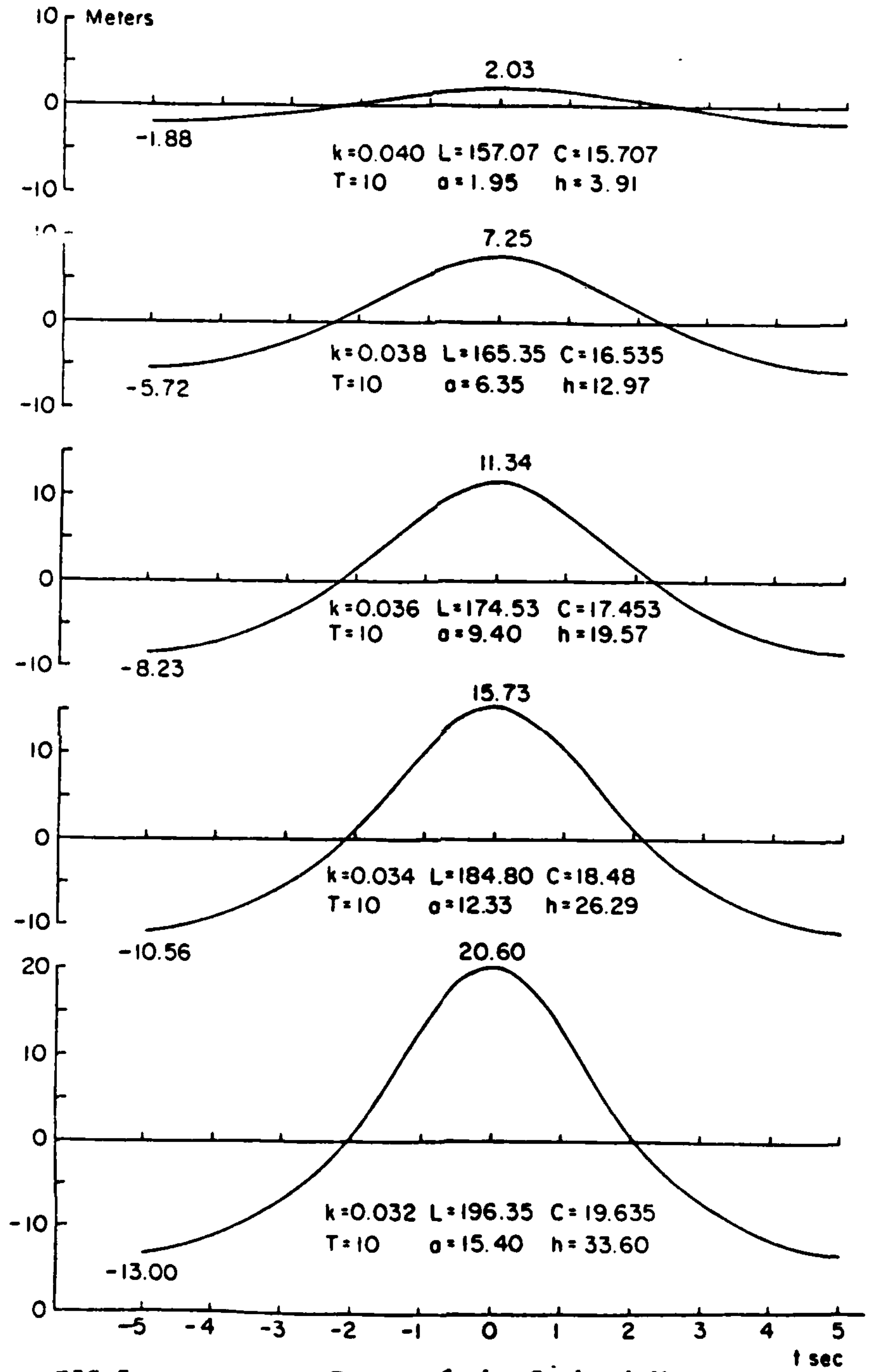


FIG 5 Nonlinear Forms of the Stokes' Wave.

The wave profile as a function of x for a frequency of $2\pi/10$ is shown in Fig. 6 along with all of the other appropriate values. The highest wave in both figures corresponds to $ak = 0.5$. As the height increases, the crest to trough assymetry increases and the wavelength increases from 157 meters to 196 meters, an increase of 25%.

There are several interesting ways to study equations (6), (7) and (8). If the amplitude of the first harmonic is increased and ω is kept constant, the wavelength and the phase speed must both increase. For a period of 10 seconds, the phase speed numerically is simply one tenth of the wavelength. The amplitudes of the first, second and third harmonics as a function of wavelength (and phase speed) are shown in Figure 7. The amplitude of the first harmonic increases rapidly as a function of wavelength at first and then more slowly whereas the third harmonic is at first negligable and then increases rapidly as the value of ak reaches about 0.5 where the curves stop abruptly.

Figure 8 shows graphs of various sums of terms that arise when equation (4) is evaluated all as a function of the value of a for a period of 10 seconds. The curve, $a = a$, is of course a line with a 45^{o} slope. The curve, $h = 2a$, would be the height of a linear wave. The curve, a_T, is the absolute value of the depth of the trough, i.e. equation (4) for $k(x - Ct)$ equal to $-\pi$. Similarly, a_C is for an arguement of zero, and $h = a_C - a_T$ is the crest to trough height. The difference between h and 2a is the result of the third harmonic, and the values of (6), (7) and (8) all depend on this being a solution to third order. The required wavelength is shown on an auxiliary scale as the value of a is varied.

Finally, the phase speed is graphed as a function of wavelength for $a\ k = 0$ and $a\ k = 0.5$ in Figure 9. The area between the two curves would show all of the possible values for phase speed and wavelength for Stokes waves if a third order pertubation expansion were good enough up to $ak = 0.5$. Lines of constant period as labled radiate from the origin.

The amplitude of the waves for the lower curve is zero. The labled points on the upper curve show the crest to trough heights of (4) for $a\ k_s = 0.5$ at the phase speed and wavelength of that point. There is a substantial range of wavelengths possible for a fixed phase speed and a considereable range of phase speed for a fixed wavelength. Quite clearly,

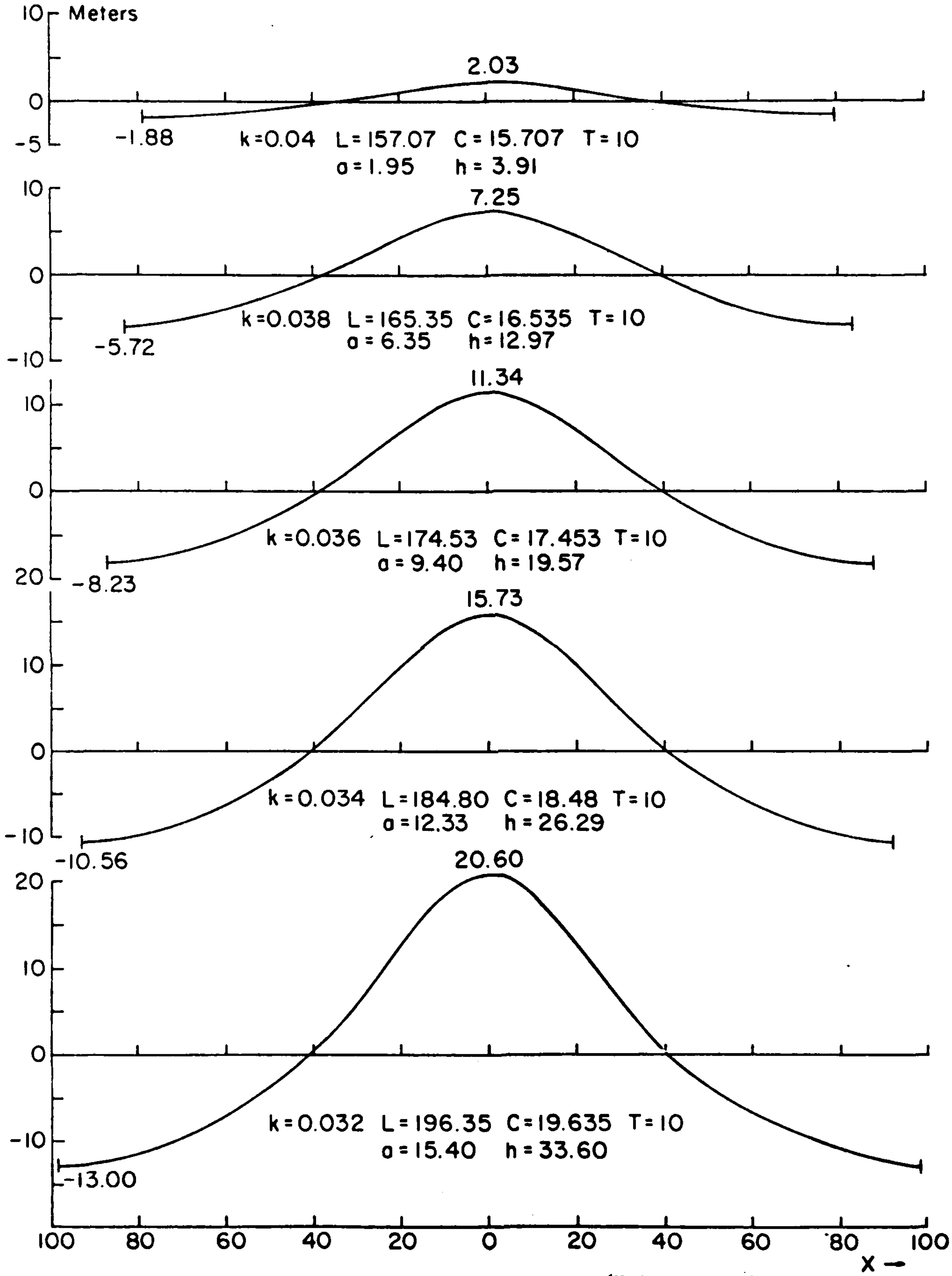
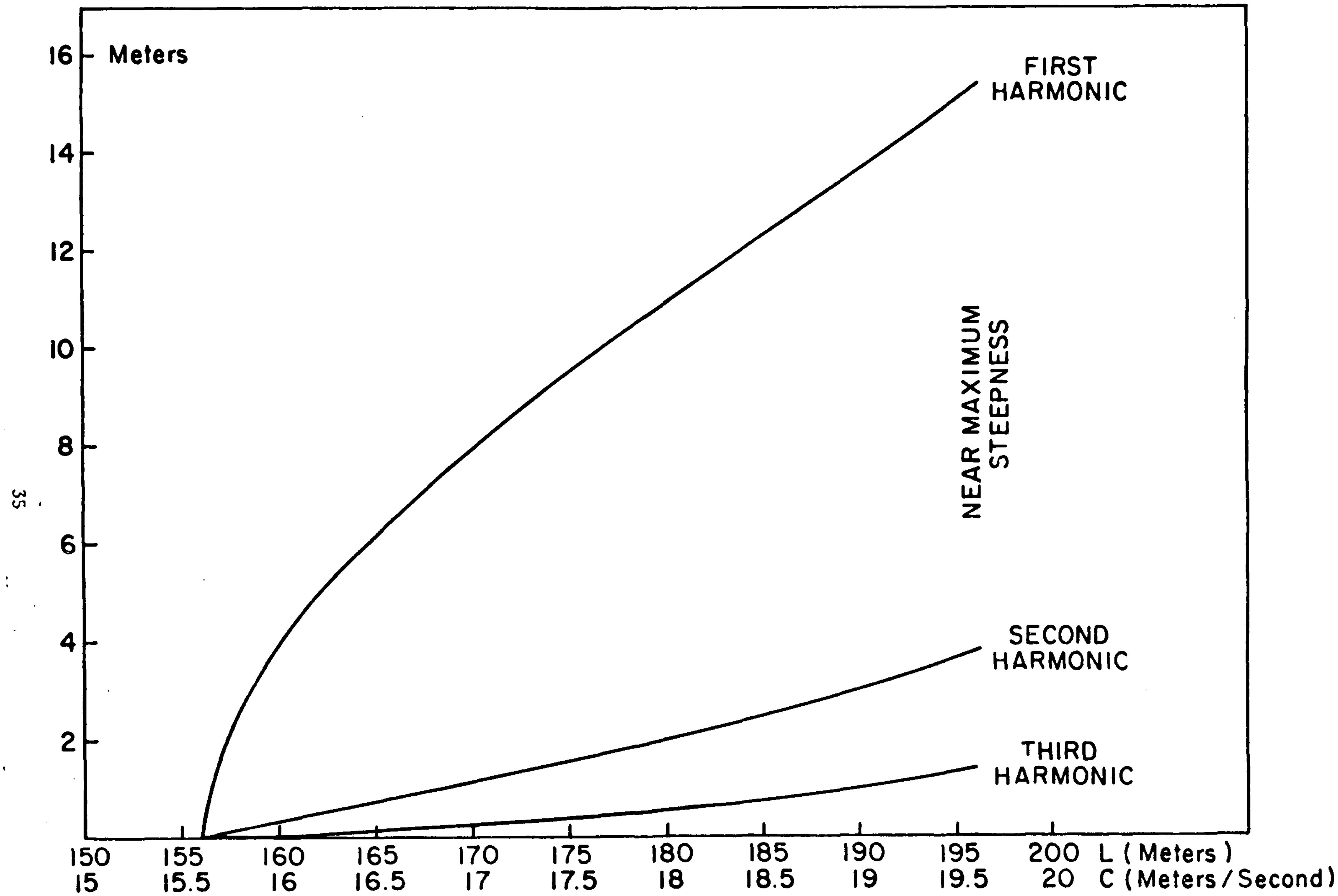


FIG. 6 Non-linear Forms of the Stoke's Wave. (X in Meters.)



FIRST, SECOND AND THIRD HARMONICS AS A FUNCTION OF WAVELENGTH (OR PHASE SPEED) FOR STOKES WAVES WITH A 10 SECOND PERIOD.

FIGURE 7

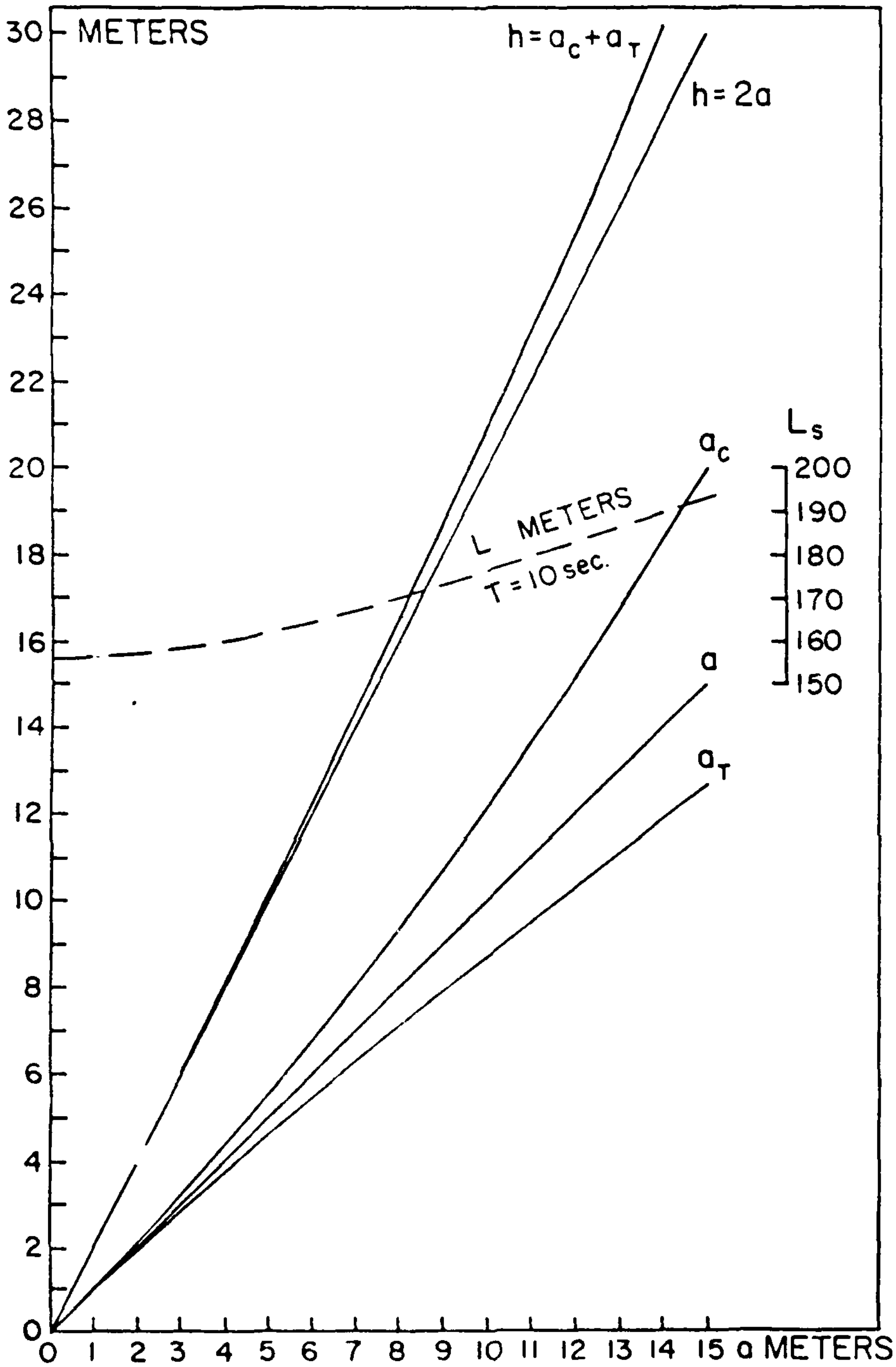


FIGURE 8 Linear Wave Amplitude, Crest and Trough Values and Total Wave Height for a 10 Second Wave as a Function of a. Dashed Curve Shows Wave Length in Meters on Right Scale.

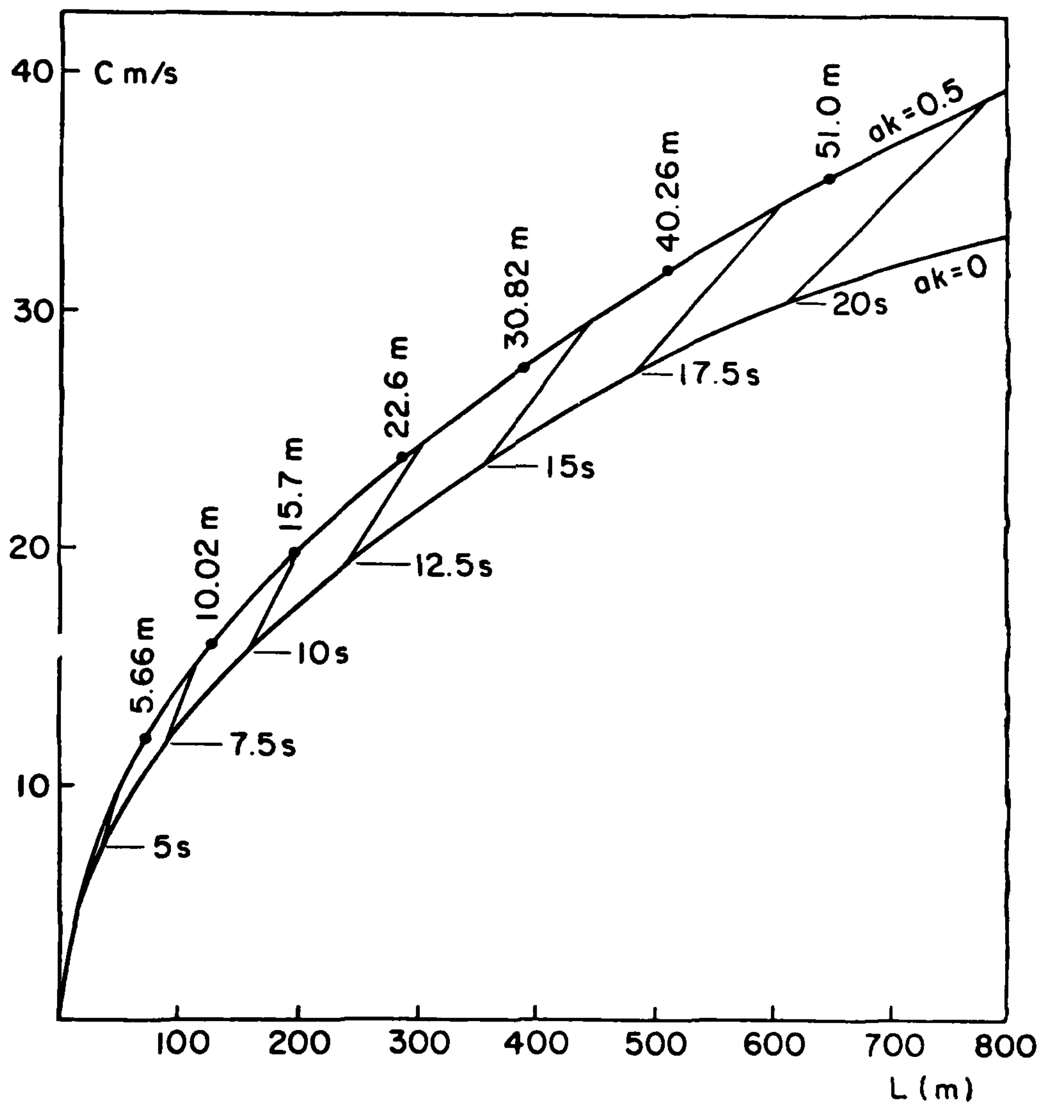


FIGURE 9 Phase Speed as a Function of Wave Length for ak= 0 and ak= 0.50 Showing Lines of Constant Period.

for this third order solution as a in (4) is increased, if k (or L) is kept constant C must increase and hence T must increase, and if kC is kept constant, T remains constant and k must decrease. When waves vary in height in a random sea, is there a corresponding effect from wave to wave? As waves in a random sea grow in height, do they lengthen and speed up? Or, as more recent results to be described later, suggest, can even stranger things happen when random waves pass through some critical value of ak locally before they break.

More Realistic Periodic Models. A third order perturbation solution for periodic gravity waves following the techniques of Stokes might be possible of generalization so as to describe random waves more realistically. Some interesting results on the non-Gaussian properties of waves have been obtained as in Huang, et al. (1983). M. S. Longuet-Higgins, and others working with him, have shown enough descrepancies between the above simple analysis and what actually happens as periodic waves increase in height to indicate that even partial success along these lines will be difficult.

For the first example, the profile of a high wave has the property that $\partial^2\eta/\partial x^2$ is positive along the entire profile except close to the crest where an angle near 120^o is formed (See Longuet-Higgins (1979a) his Figure 5, for example). The motion of the water near the 120^o corner has been investigated in detail as in Longuet-Higgins (1980). The above 1979 reference also shows that, with qualifications, the Stokes' drift for the higher waves is about three times greater near the surface than would be calculated from Stokes' original equation. At the surface, for each complete cycle of the highest wave, a water particle advances horizontally by an amount equal to 0.38 times the wavelength. Its average speed of advance is 0.27 times the phase speed. Thus, the Stokes' second order formula is not applicable for very steep waves over the whole range of depths. Some of these results are based in part on previous results of Yamada (1957) for deep water.

A very simple solution for the almost highest wave, which does not involve an infinite series and problems of convergence has been found (Longuet-Higgins (1979b)). With x positive down, y positive to the right and with

$$z = x + i\, y \tag{9}$$

$$\chi = \Phi + i\, \psi \tag{10}$$

$$\omega = u + i\, v = (d\, \chi/dz)^* \tag{11}$$

(where * is the complex conjugate) then equation (12) provides a simple, but not exact, solution for the almost highest wave.

$$\chi + i\, y = \frac{\alpha + \gamma i\, \chi}{(\beta + i\chi)^{1/3}} \tag{12}$$

For $\psi = 0$, $\chi = \phi$ and (12) gives the wave profile parametrically. In (12), $\alpha = 1.6876$, $\beta = 4.8065$ and $\gamma = 1.3104$. This approximation differs from the exact solution by at most 5% and over most of the profile by less than 1% in satisfying the pressure condition at the free surface.

Another very interesting result described by Longuet-Higgins (1975,1978a) is that the higher periodic waves are not the fastest. If C^2/gk is computed as a function of ak, with k fixed (or L), the maximum occurs at about ak = 0.436 and equals about 1.19. As a increases beyond this value, C^2/gk decreases until the highest and steepest wave is reached. The lines of constant period for a given L in Fig. 9 would require corrections to account for this property of periodic waves. According to Longuet-Higgins (1978b).

> " ----- It was generally thought that the phase-speed C of a steep, progressive gravity wave on deep water always increased with the wave steepness, at given wavelength. However, further calculations (Longuet-Higgins (1975)) have shown that the wave speed, considered as a function of the crest-to-trough height 2a, increases to a maximum within the range of possible heights, and then diminishes; the highest wave is not actually the fastest. The reason for this behaviour is associated with the fact that the profile of the highest wave, having a very sharp curvature at the crest, intersects the profile of a wave which is not quite so high. Hence the higher wave lies below it over most of the wavelength ----- . Its potential energy is therefore less, and its kinetic energy also."

Longuet-Higgins and Cokelet (1976) have studied the deformation of steep surface waves on water by means of numerical computations. Longuet-Higgins (1978b) summarized the method that was used as follows (equation and figure numbers have been changed).

> "Observations suggest that steep progressive waves have a strong tendency to become steeper on their forward face and to topple forwards, producing either "spilling" or "plunging" breakers. The development of

the wave profile, at least up to the point of impact of the jet on the forward face, has been recently followed by a new method described by Longuet-Higgins and Cokelet (1976). Although restricted in the first place to irrotational waves on deep water, the method appears to be capable of extension to waves in the presence of a surface shear current and to water of infinite depth.

"The motion is assumed periodic in the horizontal coordinate x with length scale L, though not periodic in the time t. By a simple substitution $\zeta = \exp\,[ik(x+iy)]$ the free surface is transformed into a simple closed curve C in the ζ-plane, and all points at infinite depth are transformed into a single point in the interior. By a suitable choice of coordinates the complex velocity potential $\chi = \phi + i\chi$ becomes regular and analytic everywhere inside C, at all times. The velocity field in the interior of the fluid depends only on the values of ϕ at the surface, and the computation is carried forward by following the values of x, y and ϕ corresponding to marked particles. At each time-step we compute

$$\frac{dx}{dt} = \phi_x \tag{13}$$

$$\frac{dy}{dt} = \phi_y \tag{14}$$

$$\frac{d\phi}{dt} = \frac{1}{2}\,(\nabla\phi)^2 - gy - p/\rho \tag{15}$$

where p is the surface pressure. This enables the value of ϕ on C(t+dt) to be found, and hence the tangential derivative $\partial\phi/\partial s$. To find the normal derivative $\partial\phi/\partial n$ is equivalent to solving a Dirichlet problem: How to determine the normal derivative of a function ϕ, given on the boundary and satisfying

$$\nabla^2\phi = 0 \tag{16}$$

everywhere in the interior.

" Longuet-Higgins and Cokelet (1976) find the solution by means of an integral equation, which is solved numerically, at each time step. The method involves the neglect of viscosity, compressibility and surface tension, but does not assume the particle acceleration to be small compared to g.

"In their first paper (1976) some examples were given of the development of the free surface from a symmetric wave, when the energy level was further raised by the smooth application of a surface pressure distribution, over a short initial period. After removing the pressure, the free surface curled over and plunged forward, in the way commonly observed (see Figure 10).

"In another set of numerical experiments Cokelet (1979b) has begun with a free wave of exactly sinusoidal form, but moderate or large amplitude and, without applying any pressure at the surface, has followed the subsequent development of the wave profile. Just as in the previous case the wave forms a high-velocity jet and plunges forwards (see Cokelet (1977b), Figure 10). This happens even when the initial energy is less by a factor 2/3 than the maximum energy for a steady, nonsinusoidal wave of the same wavelength ---."

"--- The momentum in the jet can grow to be a substantial portion of the total momentum of the wave train.

"To follow the jet beyond the instant where it impinges on the forward face of the wave requires a different type of analysis, which must be capable of modelling the turbulent shear flow and the entrainment of air and of momentum from the laminar part of the flow. A start in this direction has been made by Longuet-Higgins and Turner (1974)."*

The breaking waves in deep water can be followed numerically past the breaking point. Fig. 11 shows a copy, as the wave curls over, of one of the Figures in Longuet-Higgins and Cokelet (1976). Descriptions of some of the extreme waves and of ship damage at sea frequently include descriptions of events closely paralleling this kind of a wave form.

How Waves Break. In 1976, Longuet-Higgins and Cokelet (1976) wrote in the abstract, "Plunging breakers are beyond the reach of all known analytical approximations." Longuet-Higgins (1982) then proceeded to find these analytical solutions. The water need not be shallow for a plunging breaker to happen, but probably the examples that many have seen are the motion pictures of surfboard riders "in the tube" where the breaking wave has curled up and over the surfer but the plunging part of the wave has not yet fallen to the water in front of the crest.

* Our references do not give 1979a.

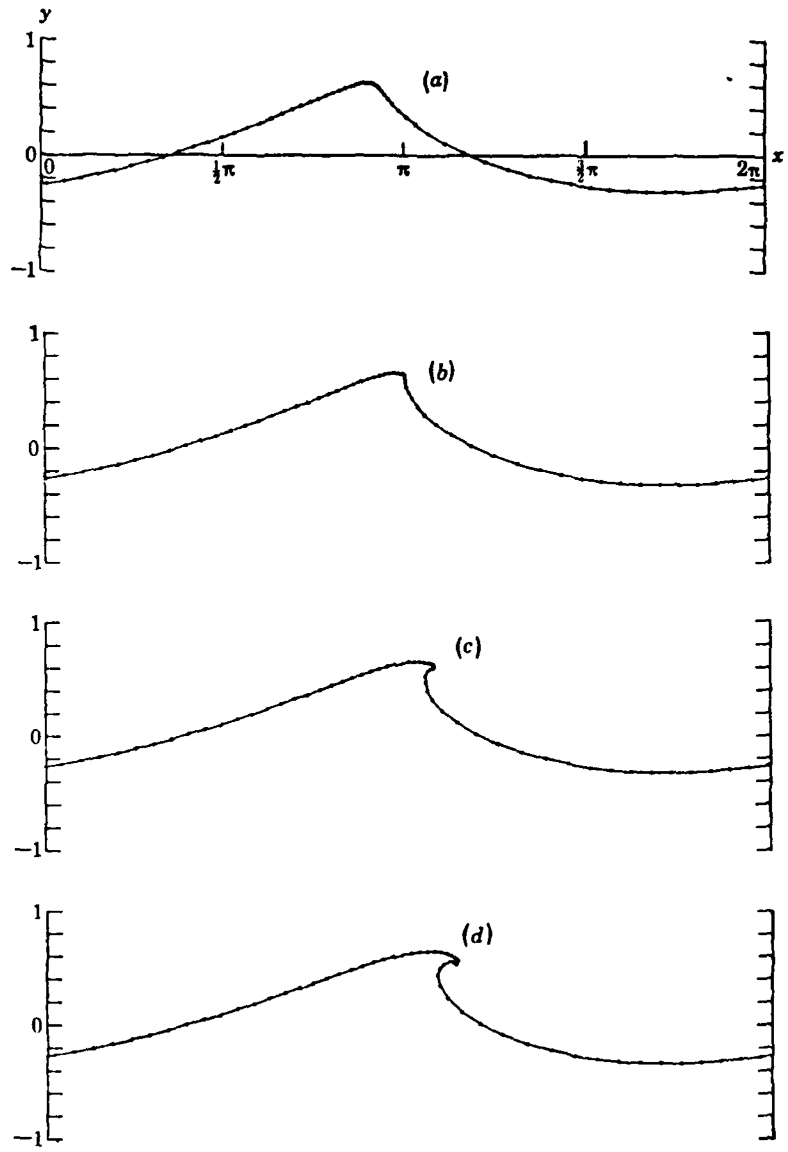


FIGURE 10 Successive Wave Profiles as a Function of (kx - ωt) for a Wave Breaking in Deep Water (From Longuet-Higgins and Cokelet (1976)).

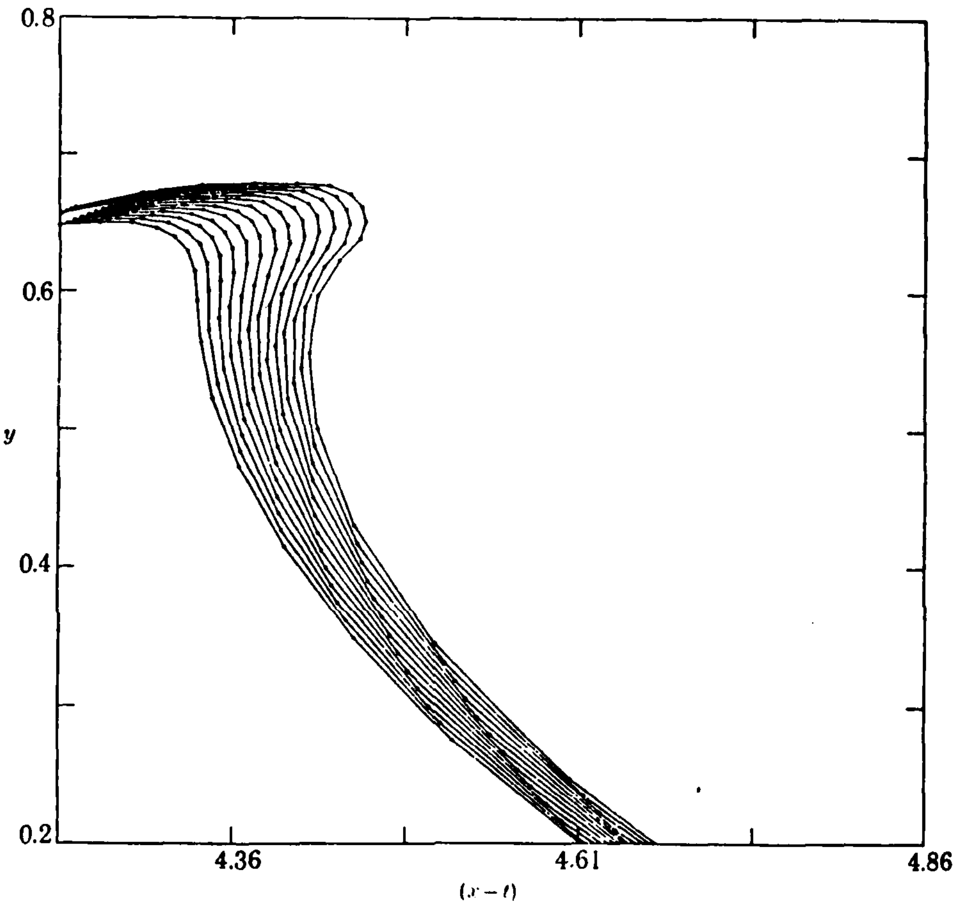


FIGURE 11 Details of a Plunging Breaker as a Function of (kx-ωt). The Points show Individual Particle Paths (From Longuet-Higgins and Cokelet (1976)).

The theory used goes back at least as far as the work of John (1952, 1953) and to previous work by Longuet-Higgins (1976). The start is with equations (9), (10) and (11), but instead of expressing χ directly as a function of z and t, it is assumed that χ and t are each analytic functions of a third complex variable ω (used differently here than elsewhere in this review) and of t. Then by eliminating ω between χ and z, it would be possible to express χ as an analytic function of z, except at a singularity. Many kinds of flows were studied, with the result that cubic flows in the parameter ω (pure imaginary) were the simplest case for which the free surface can interact itself. The singularity is inside the "tube" where there is no water and the free surface forms the inner portion of the breaking wave. The plunging part is more or less in free fall. Excellent agreement is found between the solution and the experiments of Vinje and Brevig (1981) as shown in Figure 12.

The above cited references are all based on the equations that describe irrotational flows. Most of them were concerned with periodic waves. Unfortunately, the potential function needed to describe waves extends over an infinite half space. It is thus difficult to describe steep waves near their crests and breaking and to treat irregular waves with varying heights and periods. There is, however, the possibility of piecing together parts of some of the solutions that have been found in these and other papers of the same author so as to represent storm seas more realistically.

Nonlinear Models for the Free Surface Movement of Irregular Waves. Because of the difficulties involved with irregular waves, other ways have been tried to study them. One way is to avoid using the basic hydrodynamic equations and to derive approximate equations for the motion of the free surface. These attempts appear to have begun with Zakkarov (1966) and Zakkarov and Shabat (1972). Extentions have been made by Dysthe (1979) Janssen (1983), and Hui and Hamilton (1979). A substantial review of these methods (among others) is given by Yuen and Lake (1982).

The concept involved was to show that the nonlinear behavior of waves at the free surface could be approximated by various nonlinear partial differential equations such as the Schrödinger equation. Exact solutions to this equation with a hyperbolic secant envelope have been found and called solitons.

Some extentions involve extending the equation which is supposedly

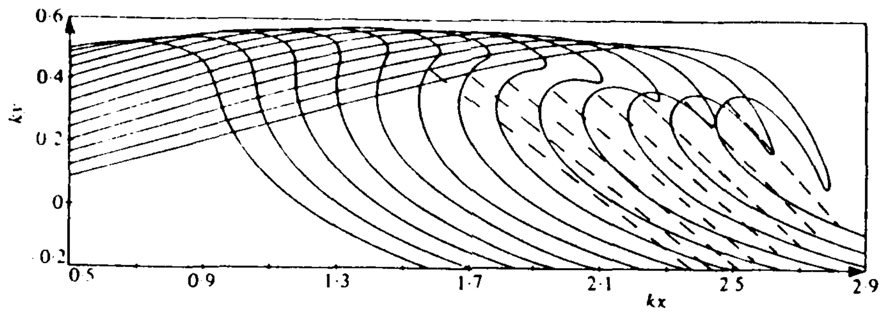


FIGURE 12 A Plunging Breaker Showing the Formation of the "Tube" in the Form of Successive Profiles (From Longuet-Higgins (1982), Based on Vinje and Brevig (1981), Fig. 4.12).

realistic at the order of $(ak)^3$ by an added term to improve it to the order of $(ak)^4$ by Dysthe (1979) and other types of solutions as in Hui and Hamilton (1979), for which the waves travel at an angle to the envelope.

These solutions have many interesting features. Some can be reproduced experimentally. Others cannot. The envelope propagates without change of shape at close to the group velocity of the carrier wave. These solutions are still under experimental study, but since solitons do not break, they do not appear to be capable of describing storm seas.

Nonlinear Dispersive Waves. Still another approach to the problem is that of Witham (1965, 1970) with further contributions by Lighthill (1967) and Yuen and Lake (1975) and a review by Yuen and Lake (1982). The concept is to treat a, k and ω (and consequently C and the group velocity) in equations patterned after equations (4), (5) and (6) as slowly varying functions of x and t with the idea that the hydrodynamic equations will be more or less locally satisfied. The additional constraints required are found by applying variational principles to the Lagrangian of the system.

For long crested waves on deep water, following Yuen and Lake (1975), the equations that result are given by equations (17), (18), (19) and (20)

$$\eta(x,t) = a(x,t)\cos\theta(x,t) + \frac{(a(x,t))^2\,k(x,t)}{2}\cos 2\theta(x,t) + \frac{3}{8}(a(x,t))^3(k(x,t))^2\cos 3\theta(x,t) \tag{17}$$

$$\begin{aligned}\phi(x,t) &= \frac{\omega(x,t)\,a(x,t)}{k(x,t)}\left\{\exp(k(x,t)z)\right\}\sin\theta(x,t) \\ &+\left(\frac{a_t(x,t)}{k(x,t)}\cos\theta(x,t) + \frac{\omega(x,t)\,a_x(x,t)}{(k(x,t))^2}\left\{1-kz\right\}\cos\theta(x,t)\right\}\cdot\exp\left\{(k(x,t)z\right\} \\ &+ \frac{\omega(x,t)\,(a(x,t))^2}{2}\sin 2\theta(x,t)\cdot\exp(k(x,t)z)\end{aligned} \tag{18}$$

where $\theta_t = -\omega(x,t)$ (19)

$$\theta_x = k(x,t) \tag{20}$$

Variation of the Lagrangian with respect to θ(x, t) gives equation (21)

$$(a(x,\ t))_t^2 + C_g(x,\ t)\ (a(x,\ t))_x^2 = 0$$

where C_g is the linear group velocity

$$C_g = \frac{1}{2}\left\{ g/k(x,\ t)\right\}^{\frac{1}{2}} \qquad (21)$$

Variation with respect to a(x, t) gives the dispersion relation, equation (22).

$$\omega(x,\ t) = \left\{gk(x,\ t)\right\}^{\frac{1}{2}}\left[1 + \frac{1}{2}(k(x,\ t)\ a(x,\ t))^2 + \frac{(a(x,\ t))_{xx}^2}{g(k(x,\ t))^2}\right] \qquad (22)$$

Finally the consistency equation is needed.

$$k(x,\ t)_t + \omega(x,\ t)_x = 0 \qquad (23)$$

These equations have a strong resemblance to equations (4), (5) and (6) with some additional correction terms. The derivations require that θ(x, t) be a rapidly varying function of space and time compared to a(x, t), hence the concept of two timing as in Witham (1970). As Witham (1965) states, "For nonlinear problems, it is assumed that exact analytic solutions corresponding to [long crested dispersive waves] are out of the question and the possibility of finding approximate solutions --- is explored." A further study of these equations will be deferred to the following section.

STUDIES OF DETERMINISTIC WAVE TRANSIENTS

Long Crested Linear Dispersive Gravity Wave Transients in Deep Water. Transient groups of waves have the property that if observed at a fixed point, the water is at first calm, then it oscillates up and down according to some pattern and then becomes calm again. If observed as a function of distance along a wave tank, for example, the disturbance and be seen to occupy a finite portion of the tank at any instant of time as it travels along. Mathematically, these features are expressed by the requirement that

$$\lim_{2T\to\infty} \frac{1}{T}\int_{-T}^{T} (\eta(x, t))^2 dt = 0 \tag{24}$$

and

$$\lim_{2\underline{\overline{X}}\to\infty} \frac{1}{\underline{\overline{X}}}\int_{-\underline{\overline{X}}}^{\underline{\overline{X}}} (\eta(x, t))^2 dx = 0 \tag{25}$$

In contrast, the above limits both approach a constant if $\eta(x, t)$ represents a stationary random process. The techniques of "Power Spectral Analysis" are therefore inapplicable for transients.

Transients can, however, be studied in terms of Fourier integrals, and the theory is well developed for a linear model. Attempted corrections for nonlinear effects were the motivation for some of the work described above.

According to Lamb (1932), (Article 236 Chapter IX), "The investigations of Arts 227-234 relate to a special type of waves; the profile is simple-harmonic, and the train extends to infinity in both directions. But since all our equations are linear (so long as we confine ourselves to a first approximation), we can, with the help of Fourier's Theorem, build up by superposition a solution which shall represent the effect of arbitrary initial conditions. Since the subsequent motion is in general made up of systems of waves, of all possible lengths, travelling in either direction, each with the velocity proper to its own wave-length, the form of the free surface will continually alter".

Suppose that a transient is observed at some arbitrary x = 0 as a function of time, and that it is an odd function. Then the Fourier spectrum is given by

$$b(\omega) = \left(\frac{1}{2\pi}\right)^{\frac{1}{2}} \int_{-\infty}^{\infty} \eta(t) \sin \omega t \, dt \tag{26}$$

The function $b(\omega)$ is an odd function and $\eta(t)$ can be chosen such that $b(\omega)$ will have a positive maximum at $\omega = \omega_o$ and a negative minimum at $\omega = -\omega_o$, with a value of zero at $\omega = 0$ as in (27),

$$b(\omega) = b_1 (\omega-\omega_o) - b_2(\omega+ \omega_o) \tag{27}$$

with $b(0) = 0$.

Then

$$\eta(x, t) = \left(\frac{1}{2\pi}\right)^{\frac{1}{2}} \int_{-\infty}^{\infty} -b_1(\omega-\omega_o) \sin ((\omega^2 x/g) - \omega t) d\omega \tag{28}$$

can represent a transient such that $\eta(0, t)$ recovers $\eta(t)$, and such that those spectral components for $\omega > 0$ are traveling in the positive x direction. As long as b_1 at $\omega = 0$ is small, the mathematical simplification of allowing some part of the disturbance to go the other way can permit the explicit evaluation of a particular model.

It is interesting to consider the Fourier spectrum of $\eta(x, t)$ as observed at some other x, say $x = x_1$ as a function of time. Equation (28) can be rewritten as (29)

$$\eta(x_1, t) = \left(\frac{1}{2\pi}\right)^{\frac{1}{2}} \left[\int_{-\infty}^{\infty} - b(\omega-\omega_o) \sin \left(\frac{\omega^2 x_1}{g}\right) \cos \omega t \, d\omega + \int_{-\infty}^{\infty} b(\omega-\omega_o) \cos \left(\frac{\omega^2 x_1}{g}\right) \sin \omega t \, d\omega\right] \tag{29}$$

where the first term is even and the second term is odd as a function of time. The Fourier transform to find the new spectrum at x_1 is, say,

$$A(\omega') = \left(\frac{1}{2\pi}\right)^{\frac{1}{2}} \int_{-\infty}^{\infty} \eta(x_1, t) \cos \omega' t \, dt \tag{30}$$

and

$$B(\omega') = \left(\frac{1}{2\pi}\right)^{\frac{1}{2}} \int_{-\infty}^{\infty} \eta(x_1, t) \sin \omega' t \, dt \tag{31}$$

The dominant term for (30) is

$$A(\omega') = \frac{1}{2\pi}\int_{-\infty}^{\infty} -b(\omega-\omega_o)\ \sin\frac{\omega^2 x_1}{g}\left(\frac{1}{2}\ \frac{\sin(\omega-\omega')t}{(\omega-\omega')}\right)dt \tag{32}$$

which reduces to

$$A(\omega') = -b_1(\omega'-\omega_o)\ \sin\frac{(\omega')^2 x_1}{g} \tag{33}$$

and similarly

$$B(\omega') = b_1(\omega' - \omega_o)\ \cos\left(\frac{(\omega')^2 x_1}{g}\right) \tag{34}$$

so that

$$(A^2 + B^2)^{\frac{1}{2}} = b_1 \tag{35}$$

and the Fourier integral spectrum has a certain invariant property as the disturbance travels as a function of x.

It is usually not too difficult to pick an $\eta(t)$ such that (26) can be integrated, but the integration of (28) is often complicated. As a last resort, the stationary phase approximation can be used (see, for example, Hui (1983)) so that

$$\eta(x,\ t) \sim \sqrt{\frac{g}{2x}}\ b\left(\frac{gt}{2x}\right)\ \sin\left(\frac{gt^2}{4x} + \frac{\pi}{4}\right) \tag{36}$$

Two examples are known for $\eta(t)$ picked in such a way that both (26) and (28) can be integrated. One is equation (37) (See Neumann and Pierson (1966)).

$$\eta(t) = -a\ e^{-B^2t^2}\ \sin\omega_o t \tag{37}$$

which yields

$$\eta(x,t) = \frac{-a}{D^{\frac{1}{2}}}\exp\left(-\left(\frac{4\omega_o^2\ B^2}{g^2 D}\right)\left(x - (gt/2\omega_o)\right)^2\right)$$

$$\cdot\ \sin\left(\frac{\omega_o^2\ x}{gD} - \frac{\omega_o t}{D} - \frac{4B^4 t^2 x}{D\ g} + \frac{1}{2}\tan^{-1}\frac{4B^2 x}{g}\right) \tag{38}$$

where $D = 1 + (16\ B^4\ x^2/g^2)$ (39)

In a linear theory,

$$\frac{\partial \phi}{\partial t} = g\eta \quad \text{at } z = 0 \tag{40}$$

so that the time derivative of the potential function can be found ($\Phi_{xx} + \Phi_{zz} = 0$) by integrating

$$\frac{\partial \Phi}{\partial t} = \frac{1}{(2\pi)^{\frac{1}{2}}} \int_{-\infty}^{\infty} - b(\omega - \omega_o)\, e^{\frac{\omega^2}{g} z} \sin\left(\frac{\omega^2 x}{g} - \omega t\right) d\omega \tag{41}$$

If this is done, the term $x - (gt/2\omega_o)^2$ becomes

$$z^2 - \frac{g\ z}{4B^2} - \frac{B^2 g\ \pi^2\ zt}{\omega_o^2} + (x - (gt/2\omega_o))^2$$

the term D becomes

$$D = 1 - \frac{8B^2 z}{g} + \frac{16B^4\ (z^2 + x^2)}{g^2}$$

and $\frac{1}{2} \tan^{-1} 4B^2\ x/g$ becomes

$$\frac{1}{2} \tan^{-1} \left(4B^2\ x/g(1 - (4B^2\ z/g))\right).$$

These changes account for the fact that the effects of the shorter waves decrease more rapidly with depth, z, (positive up and not x + iy as before).

A second solution for

$$\eta = -a \sin \omega_o t \tag{42}$$

if $-NT_o < t < NT_o$ for integer N and zero otherwise, where $T_o = 2\pi/\omega_o$, can be found in terms of Fresnel integrals (See Kinsman (1965)). For n small, the evaluation is messy, and it is rather surprizing how well (36) seems to fit the solution for low waves that have traveled a short distance. The envelope, b(gt/2x), has a form that seems to fit the transients given by Yuen and Lake (1982) (pg. 81) and Su (1982) (Figs. 2, 3 and 4 for large fetches) quite well.

Linear Dispersion and Coalescence. The solutions to the kind of problem described immediately above are supposedly well known. If near some x, $\eta(x,t)$ takes a brief time to pass at $x = 0$, then the waves disperse, or spread apart with distance traveled so that visibly longer waves with longer apparent periods arrive at a given point first, followed by moderately long waves and moderate apparent periods and finally by the shorter waves with shorter apparent periods. What is probably also well known, but not usually recognized, is that waves can un-disperse, or coalesce. For example, if (38) is evaluated at some moderately large negative value of x as a function of time, short apparent period waves are seen first, followed by moderate apparent period waves with the long apparent period waves coming along last. As the coalesing group moves along, the waves all combine to produce (37) at $x = 0$.

Some Wave Tank Experiments. The concept of generating a coalescing wave transient was described by Cummins (1962). Periodic waves of varying heights and random seas were two available ways to determine the response of model ships to waves. The idea that a transient would contain a wide range of wave frequencies, and yet, in a sense, be deterministic, was suggested as a way to reduce the number of test runs of a particular model.

The mathematical aspects of the analysis were difficult, high speed computers were not quite high speed enough, and the concept of an FFT would not be available for at least three more years (Cooley and Tukey (1965)). With all of the inherent difficulties, Cummins (1962) had the insight to realize that something new might be learned by studying the transient response of a model ship to a passing wave transient.

Davis and Zarnick (1964) persued this line of investigation in 1962 and 1963. Their figures (Figs. 6, 12, 27, 28) have considerable similarity to our Figures 13 to 16. The theories used in both of these studies were linear theories whereas as the transient coalesces the waves in it become more and more decisively nonlinear. Although a linear theory yields the same Fourier integral spectrum for a transient everywhere except for phase shifts, the nonlinear effects are quite different and a model encountering the transient during different stages of its coalescence would behave quite differently.

At Hydraulics Laboratory of the National Research Council of Canada and the Hydraulics Laboratory of the Canada Centre for Inland Waters, experiments in which reproducable wave transients can be generated and studied have also

been done. These experiments can help bridge the gap between nonlinear periodic waves and their properties and the random nonlinear wave found in nature.

Mansard and Funke (1982) needed to test models of communication buoys and various structures for the effects of waves that break over them, or around them. They developed a way to generate transients at the wave board (including a model for the board transfer function) such that the waves would travel down the tank and coalesce as they advanced. At the test object, a plunging breaker (See the three photographs in the reference) could be formed as well as various forms of isolated short duration wave groups. They were different from the above linear theory only in that, if they did not break, they were very nonlinear. Fig. 13 shows the input wave form at a distance of 1.8 m from the wave generator, relative to an arbitrary zero as redrawn from the reference. The oscillations from zero to 20 seconds are too rapid to redraw well and so only the envelope is shown as the apparent periods decrease to the left. The reproducable output from the generator would be described in engineering term as an amplitude modulated frequency modulated chirped signal. The input at 1.8m, after traveling 21 m changes into the second wave form. The highest crest to trough wave in the input wave form is roughly 0.15 m. At 22.8 m, the height is 0.28 m (almost double). Referred to by the authors as an episodic wave, the mechanism of its formation is the coalescence of a dispersed train of waves with short waves in front and longer waves behind that could conceivably occur at random times and places in an actual seaway.

Figure 14 illustrate attempts by Mansard and Funke (1982) to generate various wave forms at a measurement point 22.8 m from the generator. The left hand column of figures shows what was desired, and the right shows what was measured. For the first three, the mean to crest and the mean to trough desired signals had the same value. The measured signal had higher crests and shallower troughs. High waves will not behave as linear waves. The last desired transient was made nonlinear, and the output differs from it in peculiar ways. The third measured transient looks more like the fourth desired transient than the fourth measured transient.

A study of transients produced by a generator capable of moving in the horizontal direction exactly as programmed by a time varying signal is being

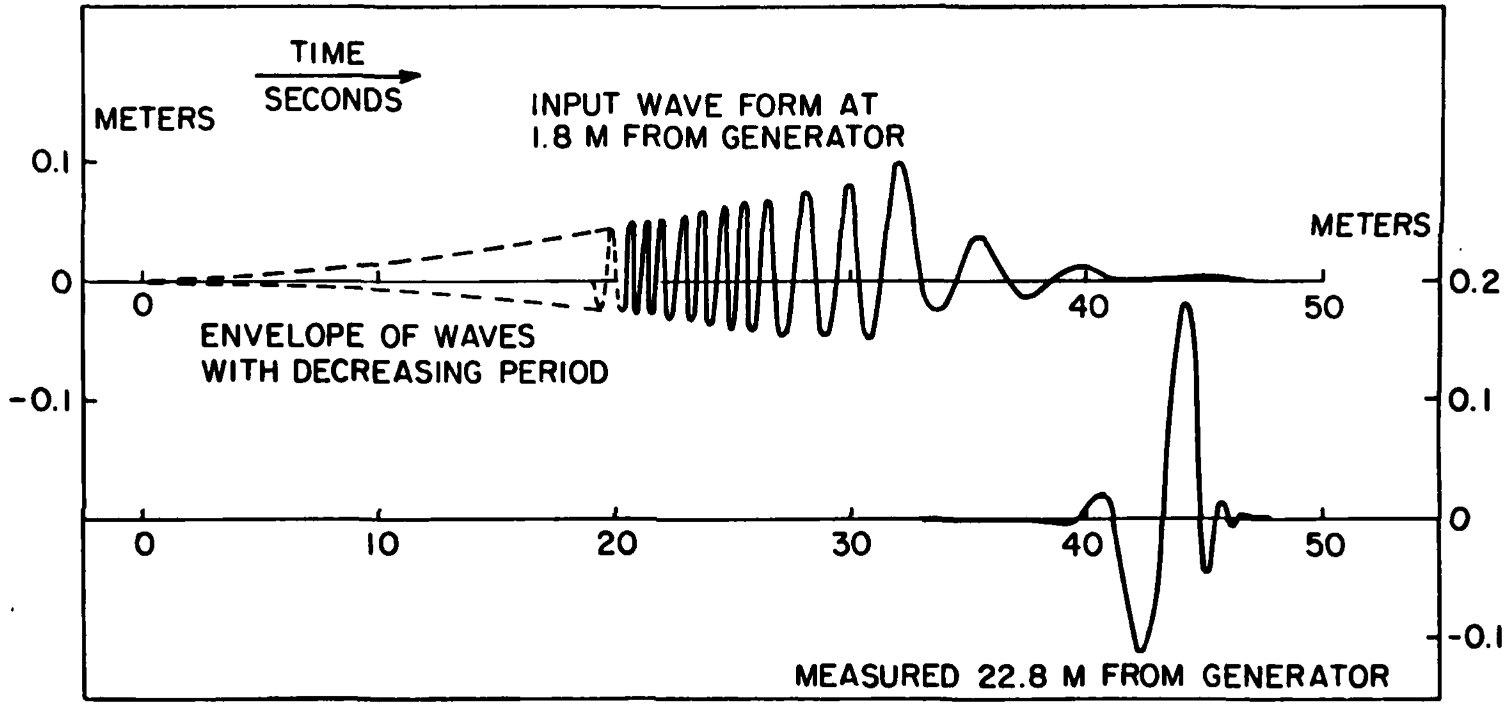


FIGURE 13 Transient Waves Reported by Mansard and Funke.

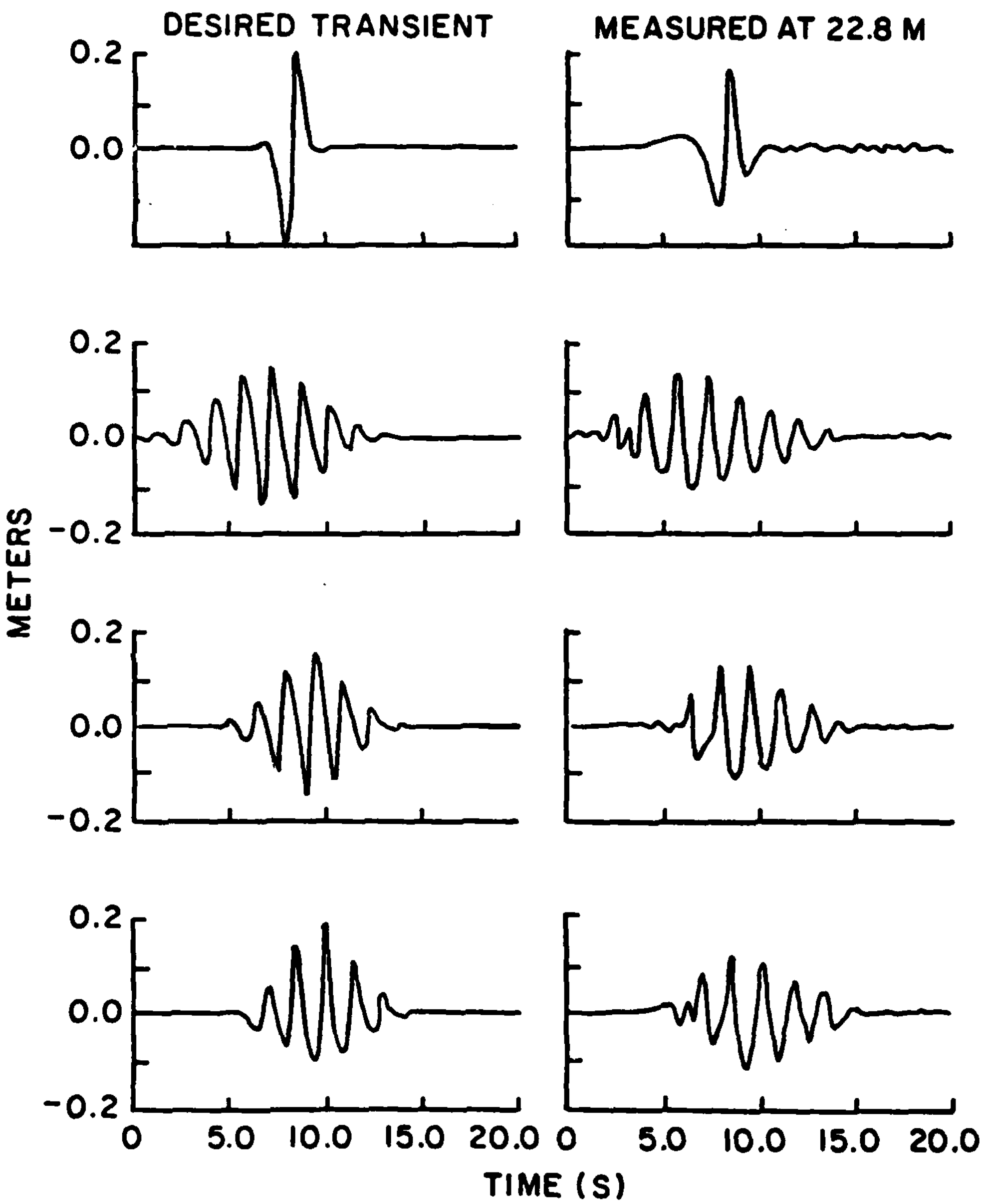


FIGURE 14 Desired and Measured Transients Obtained in a Wave Tank. Redrawn from Mansard and Funke (1982).

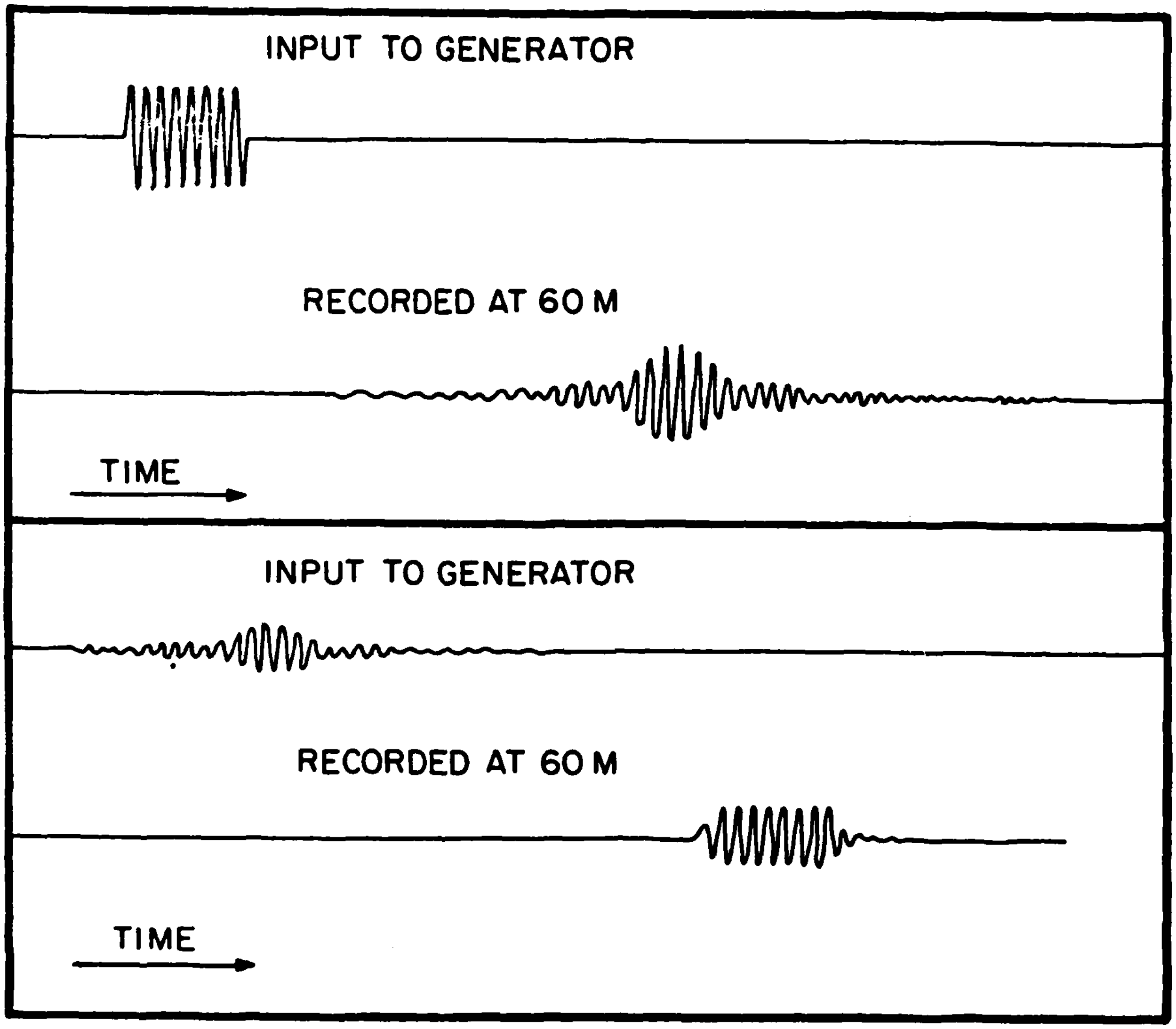


FIGURE 15 Inputs to a Wave Generator and the Resulting Wave Transients Measured at the Canada Centre for Inland Waters. The Second Input is the Time Reversed Measurement of the Waves Recorded at 60 m for the First Input.

done by M. Donelan, W. H. Hui and W. Pierson. Results are only in a preliminary stage, but we plan to study both low transients and transients with increasing amplitude. It is possible to check some of the results of the various theories described previously. There is a transfer function between the motion of the generator and the resulting wave motion so that the vertical scales are not comparable, nor are the scales for the recorded data. The gain was changed from one run to another to optimize the digital records. The general shapes for the inputs and the outputs are comparable. The time origins are not comparable from one pair to another.

The top part of Figure 15 shows a sine wave with a box car envelope that is turned on instantaneously at the start and off instantaneously at the end. There are eight oscillations under the box car. It is difficult to do this with a generator and the response of the water in front of the generator must be complicated. The waves 10 m from the generator probably do not have a spectrum that relates to the input function very well. The amplitude of the input was low. However, for the waves recorded at 60 m, instead of eight there are quite a few more. Long apparent period waves arrive first, and so on. The envelope has a main maximum, preceded and followed by two smaller groups, which in turn may be preceded and followed by two still smaller groups.

The waves that passed the wave recorded at 60 m were recorded on an FM tape recorder, and the result was played back into the wave generator with the time reversed. The waves recorded this second time around have coalesced and almost, but not quite, reformed the first input signal, except that it is upside down. The sign of the time reversed signal should have been changed. The first trough is a little too small but the next seven troughs are all nearly equal. The first crest is an extra one, but it is followed by eight crests of nearly equal height and these are trailed by a small trough, a smaller crest, another small trough and some wiggles.

Although the second record is not an exact duplicate of the original input to the generator, it comes close with differences probably explainable by the form of the input function and the generator transfer function.

Figure 16 shows an input signal based on equation (37). The chart speed was doubled to get a good look at the function so that it has to be bunched up to half its present length when comparing it to the other curves.

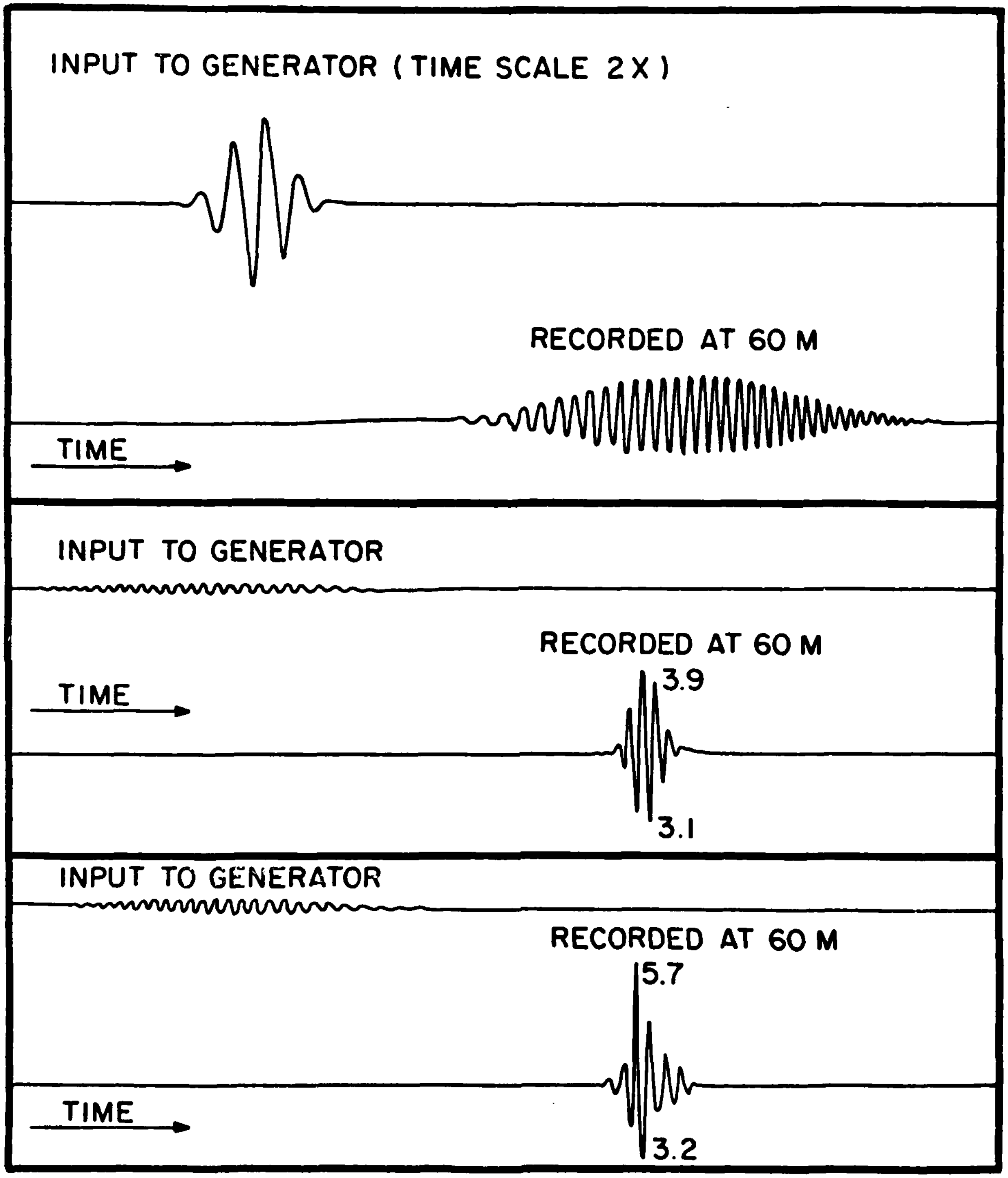


FIGURE 16 Input to a Wave Generator and the Resulting Wave Transients Measured at the Canada Centre for Inland Waters. The Second Two Inputs are the Time Reversed Measurements of the Waves Recorded at 60 m for the First Input Except for Different Gains at the Generator.

There are five crests and five troughs. The generator was driven with a low amplitude signal. By the time the waves had traveled 60 m, they had dispersed with one smoothly varying envelope, with, of course, long apparent periods at front and the short ones at the rear. For this example, the signal at 60 m was played back into the generator with time reversed, but for one run slightly amplified and for the second run substantially amplified. For the slightly amplified input, which was still quite low and nominally nearly linear, the highest crest was 3.9 and the lowest trough 3.1 chart scale units. The coalesced record had become nonlinear. The chart scale units suggest a value of $ak \simeq 0.2$ had the wave been one of constant wave-number. For the moderately nonlinear record, there were a few extra low oscillations, but the original input was nearly recovered.

With an even higher, but still time reversed signal, the record at 60 m was highly nonlinear ($ak \cong 0.5+$). The waves did not break, but the recorded waves were quite distorted compared to the input. Further amplification of the input resulted in one wave in the group breaking at an ex post facto predictable location upon repetition in the wave tank.

In a random seaway, every once in a while a train of waves can form that are traveling in just the right direction with increasing apparent wave length and period in a direction opposite to that of the moving waves. It is then possible for this group of waves to coalesce into one exceptionally high wave, or more, that seems to form out of nowhere. Fortunately, they are rare and can break and disperse almost as quickly as they form. They might be described as "episodic" but they are surely not "freaks" because they are explainable, to a large extent, by known properties of gravity waves and they can be formed at will experimentally in the laboratory.

The SNAME review (Seakeeping 1953-1973) cites, the papers by Cummins (1962) and Davis and Zarnick (1964) in reviews by Ogilive and Beck (1974) and Ochi (1974). However, Dalzell (1974) reviewed the area of model testing in random seaways and notes both his (and others) surprize that the linear model seemed to work quite well even in extreme seaways. The details of an extreme event (that does not cause capsizing as in Paulling and Wood (1974)) may be lost in the computational procedures used to analyse a random process.

Ever higher periodic waves are completely unrealistic. Perhaps something can be learned about the nature of the catastrophic events reported by men of the sea by co ining the right kinds of transients with a random seaway and making sure that the model will be in the right place at the right time (or if one prefers the wrong place at the wrong time for survivability).

<u>A Nonlinear Transient Wave Packet</u>. The solution of (37) given by (38) might somehow be related to equations (17) to (23) with a little care in changing cosines to sines. However, the argument of the sine in (38) when used to calculate a local wavenumber and frequency from (19) and (20), say $\tilde{k}$ and $\tilde{\omega}$, will not give a local progressive wave of the form

$$\eta(t) = a \sin (\tilde{k}x - \tilde{\omega}t) = a \sin \theta \tag{43}$$

Also, the θ in (17) must be a function of the local amplitude of the wave train which involves the exponential term in (38), and $\theta_t = -\tilde{\omega}$ plus $\theta_x = \tilde{k}$ must also satisfy (22). Whatever phase function results must also satisfy (38) when the amplitude of the disturbance goes to very small values. Finally, the expression for the potential function in (18) does not provide for the added complexity that k also varies with depth as in (41).

Nevertheless, it is interesting to try to see how close one can come to an extreme wave by studying a coalescing and then dispersing transient. To do this, we consider the transient to be a nonlinear improvement on (38) and study its behavior as a function of time at various fixed values of x, designated by $x = x_1$. The envelope in (38) is a maximum as a function of time when $t = t_o = 2\omega_o x_1/g$ so that $t = t_o + t'$ can be a transformation of the time to a value when the peak of the transient passes, $x = x_1$. Thus,

$$t = \frac{2\omega_o x_1}{g} + t' \tag{44}$$

and

$$t - t_o = t' \tag{45}$$

The argument of (38) reduces to equation (46)

$$\theta = - \frac{-\omega_o^2 x_1}{g} - \left(\omega_o + \frac{4B^4 x_1}{D(x_1)g} t'\right)t' + \frac{1}{2} \tan^{-1}\left(4B^2 x_1/g\right) \tag{46}$$

so that the local frequency is

$$\tilde{\omega} = \omega_o + \frac{4B^4 x_1}{D(x_1)g} t' \tag{47}$$

The value for a(x, t) in (17) becomes

$$A = a(x,t) = \frac{-a}{(D(x_1))^{\frac{1}{2}}} \exp \left(-(4\omega_o^2 B^2 (t')^2)/g^2(D(x'))\right) \tag{48}$$

and all that is needed is $\tilde{k}$ (x, t).

If it is assumed that (47) and (48) are correct, then (22) can be solved numerically for $\tilde{k}(x,t)$, and (17) can be evaluated as a function of t' for a fixed x_1. The phase function thus depends on the local amplitude of the envelope as the transient passes by.

For $\omega_o = 2\pi/1.5$, $B^2 = \frac{1}{3}$ and a convenient value for a in the linear model, t_o can be varied in 0.25 second steps to find x_1, and t' can then be varied in 0.25 second steps to generate time histories at each x_1. When referred back to true time t, the values for the same time at successive values of x_1 generate the wave profile as a function of x.

Figures 17a and 17b show the wave profiles as a function of x as time varies from -4.25 seconds to 4.00 seconds. The transient is moving from left to right. Much farther to the left, the transient would be a sequence of many low waves with long waves to the rear and short waves in the front as in previous figures. At -4.25 seconds there is the crest of the long wave at -8 meters, a trough at about -5.5 meters a crest at -4.4 meters, a trough at -3 meters and a small crest near -2 meters. The transient has not reached x = 0.

As time passes the rear wave grows in height and shortens in length until -2.00 seconds. It is followed by a trough that gets deeper and deeper until -0.75 seconds, and as the trough forms a crest begins to build up behind it.

The deepest trough forms at -0.75 seconds with a value of -0.25 m, and if scaled up by Froude's law by 10 so that ω_o was $2\pi/15$, the result would be a fairly realistic "hole" in the sea as described in the introduction.

The wave form is nearly an odd function of x at t = 0 in Figure 17b except that the crest is higher and the trough shallower. (Perhaps -0.25 is even closer to an odd function).

The crest that started to build at about -1.0 seconds reaches its peak at 0.75 seconds with a value of 0.42 m. It rapidly progresses through the envelope decreasing in height until it is nearly gone and quite long at 4.00 seconds.

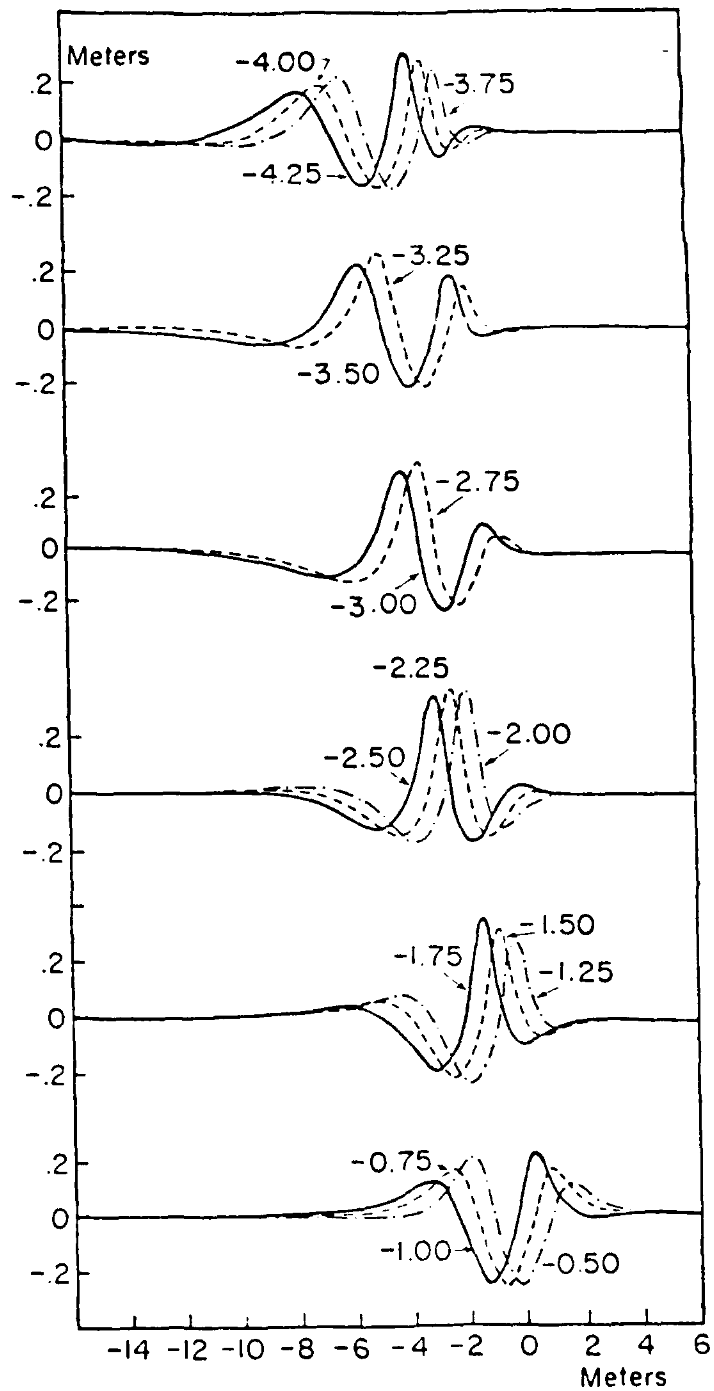


FIGURE 17a (See 17b).

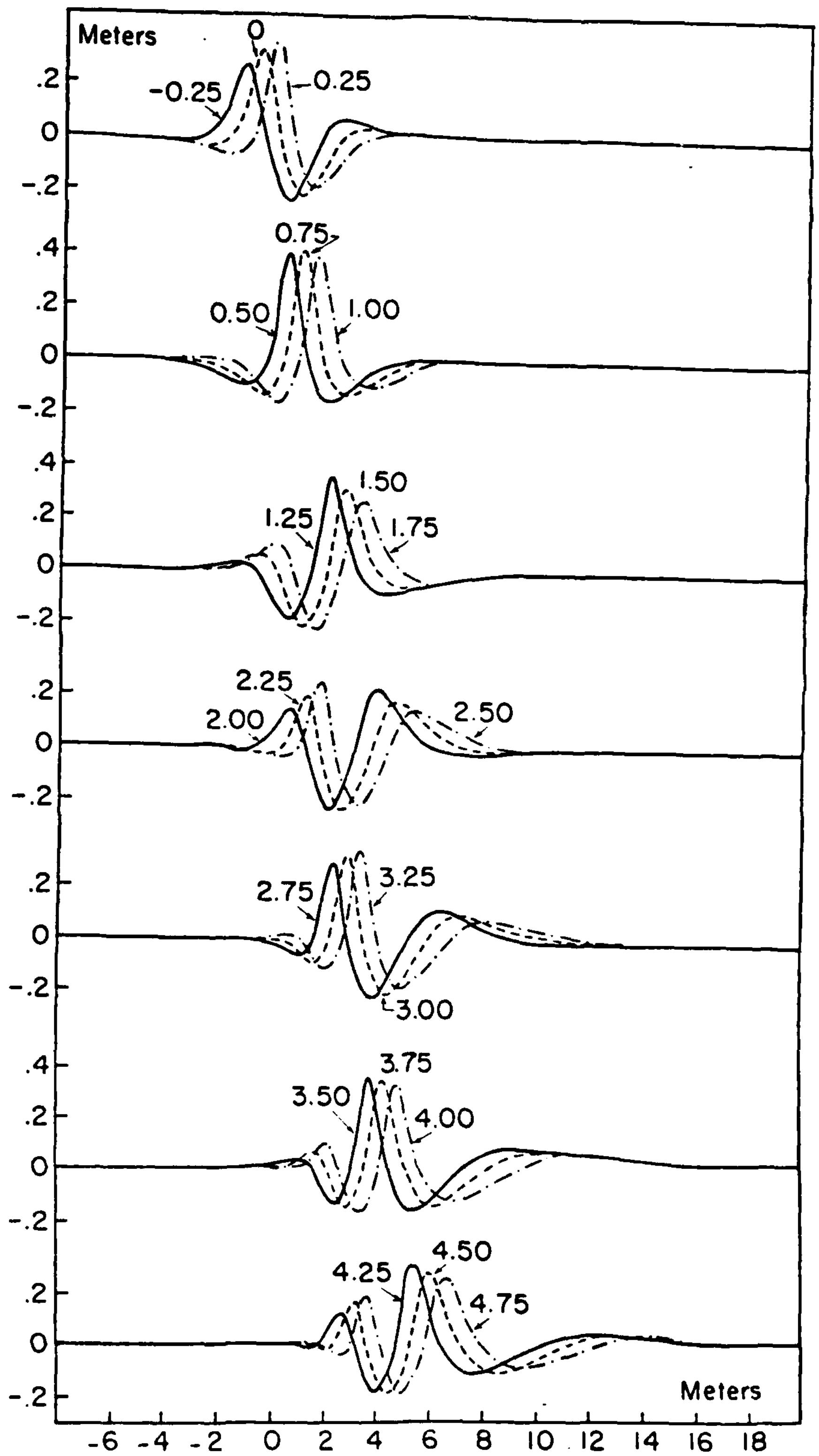


FIG. 17b Successive Wave Profiles as a Function of x at Different Times as a Transient Passes x = 0.

Near 1.25 seconds another crest starts to form, increases in height until 3.25 seconds and then starts to shrink. It is never as high as the crest at 0.75 seconds. The end of the sequence at 4.00 seconds look very much like the mirror image of the -4.00 profile near the start of the sequence. Long waves are now at the front and short waves are at the rear. The wave form at 0.75 seconds looks like the time histories that Buckley (1983) gives for Hurricane Camille.

The analysis used here is probably not quite right, although the envelopes of transients are frequently propagated in recent papers at the group velocity of the carrier frequency as in (38). Had equation (21) been used, the envelope would become distorted and a nonlinear function of itself. The group velocity would increase with increasing wave amplitude, and it would be difficult to keep track of time and space reference points. The results obtained are considered to be close, but not close enough, to reality.

THE PROBABILITY DENSITY FUNCTION FOR THE CREST TO TROUGH HEIGHTS OF OCEAN WAVES

Review. The significant wave height was originally defined in terms of visual observations. When waves were first recorded instrumentally as a time history, a relationship between the visual observations and the waves in a time history was needed. The definition that seemed to fit best was that the significant wave height was the average of the one third highest waves in a time history as recorded. The significant period was the average of the periods of these one third highest waves. The beginning attempts at wave forecasting tried to predict these two numbers.

Upon the development of spectral analysis procedures, the work of Rice (1944), which is valid for any linear, hence Gaussian, process showed that the envelope of a Gaussian process has a Rayleigh pdf. For a narrow band process, a quick extention to the pdf of crest to trough wave heights was made as in (1).

The moments of an ocean wave spectrum are defined, depending on the author, as

$$E_j = m_j = \mu_j = \int_o^\infty \omega^j S(\omega)d\omega \tag{49a}$$

for $j = 0, 1, 2, 3$ and 4, and such that for $j > 4$, the concept of a moment for waves starts to break down because one is looking at high frequencies beyond the range of most wave recorders and various kinds of noise can contaminate the calculation from spectra estimated from actual data.

Given f(h) in (1), the value h_1 can be found such that

$$\int_{h_1}^\infty f(h)dh = \frac{1}{3} \tag{49b}$$

$$\text{Then } h_{\frac{1}{3}} = 3\int_{h_1}^\infty hf(h)dh = 4.0046(E_o)^{\frac{1}{2}} \tag{50}$$

Since E_o is not very well known from a typical wave record, the equation can be simplified to (3).

Difficulties begin to arise when the same wave time history is processed in two diffirent ways; one by ranking the waves from the lowest to the highest

and averaging the heights of the one third highest waves and the other by estimating the spectrum, integrating it to get E_o, and computing $h\frac{1}{3}$ from (3). If the first way yields a value that is nearly aways less than the second as in Figs. 1, 2, and 3, there is reason to question the validity of the Rayleigh pdf. Even though the confidence limits on the estimate of the significant wave height from the estimated spectrum is usually broad enough to include the significant wave height computed from the actual time history, the fact that the two are different with one nearly always lower than the other for the same time history is the source of the problem.

The difficulty is not only with reference to the significant wave height but also can affect the value of the average of, say, the one tenth highest waves and the expected value of the highest wave in a sample of size, N. Only the last of the three ways to study extreme wave heights described by Ochi (1982) would remain valid, and the data base for this method is limited and not of very good quality.

Past Results. The crest to trough wave height pdf has been studied by Longuet-Higgins (1952, 1980) Spring (1978) Forrestall (1978) Tayfun (1981a,b,1983) and Pierson and Salfi (1982) (unpublished report for DTNSRDC), among others.

An ocean wave time history as a Gaussian (and hence) linear process can be represented by

$$x(t) = \eta(t) = R(t) \cos \theta(t) \tag{51}$$

Instead of x(t) in a Taylor series about t = 0, one can as in Pierson and Salfi (1982) follow Ehrenfeld, et al. (1958) and express R(t) and θ(t) in a Taylor series about a time picked at random as in (52).

$$\eta(t) = \left(R_o + \dot{R}\,t + \ddot{R}t^2/2\right) \sin \left(\theta_o + \dot{\theta}t + \ddot{\theta}t^2/2\right) \tag{52}$$

It was possible to find the multivariate pdf of the six random variables in (52) as, say,

$$f(R_o,\ \dot{R},\ \ddot{R},\ \theta_o,\ \dot{\theta},\ \ddot{\theta})$$

which upon integration over the domain of definition yields one.

The treatment of the $\ddot{\theta}$ term in the pdf above was intractable and (52) was simplified to (53) by integrating out $\ddot{\theta}$.

The above pdf would have to be evaluated over a five dimensional space to generate portions of equation (52).

$$\eta(t) = (R_o + \dot{R}t + \ddot{R}t^2/2)\ \sin\ (\theta_o + \dot{\theta}t) \tag{53}$$

If $\theta_o = 0$ (or by a similar analysis π)*, a crest will occur at $\dot{\theta}t = \pi/2$ so that

$$\eta\left(\frac{\pi}{2\dot{\theta}}\right) = R_o + \dot{R}\pi/2\dot{\theta} + \ddot{R}\ \pi^2/8\dot{\theta}^2 \tag{54}$$

A trough will occur at $\dot{\theta}t = -\pi/2$ so that

$$\eta\left(-\frac{\pi}{2\dot{\theta}}\right) = -\left(R_o - \dot{R}\ \pi/2\dot{\theta} + \ddot{R}\quad \pi 2/8\dot{\theta}^2\right) \tag{54}$$

The height of the wave will be

$$h = 2\ R_o + \ddot{R}\ \pi^2/4\dot{\theta}^2 \tag{56}$$

Equation (56) shows that h is not dependent on $\dot{R}$ to this level of approximation. If the above pdf is integrated over $\dot{R}$ first, and then $\ddot{\theta}$, the result is equation (57).

$$f(R_o,\ \ddot{R},\ \dot{\theta}) = \frac{K^{-1}\ R_o^2\ \dot{\theta}}{2\pi\ \Delta^{\frac{1}{2}}\ E_o^{\frac{1}{2}}} \exp -\tfrac{1}{2}\left(\frac{\psi}{E} R_o^2 + \frac{E}{\Delta}(\ddot{R} - (\dot{\theta}^2 - \frac{(D\dot{\theta} + F)}{E}) R_o)^2\right) \tag{57}$$

The transformation of variables

$$R_o = \left(\frac{E}{\psi}\right)^{\frac{1}{2}} \delta \tag{58}$$

$$\ddot{R} = \left(\frac{\Delta}{E}\right)^{\frac{1}{2}} \gamma + \left(\dot{\theta}^2 - \frac{D\dot{\theta} + F}{E}\right)\left(\frac{E}{\psi}\right)^{\frac{1}{2}} \delta \tag{59}$$

$$\dot{\theta} = \dot{\theta} \tag{60}$$

whose inverse is

$$\delta = \left(\frac{\psi}{E}\right)^{\frac{1}{2}} R_o \tag{61}$$

$$\gamma = \left(\frac{E}{\Lambda}\right)^{\frac{1}{2}} (\ddot{R} - (\dot{\theta}^2 - \frac{D\dot{\theta} + F}{E}) R_o) \tag{62}$$

*With crest and trough interchanged.

$$\dot{\theta} = \dot{\theta} \qquad (63)$$

yields equation (64) from equation (57) and equation (65) from equation (56).

$$f(\delta, \gamma, \dot{\theta}) = \frac{2\delta^2 e^{-\frac{1}{2}\delta^2}}{(2\pi)^{\frac{1}{2}}} \cdot \frac{1}{(2\pi)^{\frac{1}{2}}} e^{-\frac{1}{2}\gamma^2} \cdot \frac{K^{-1} E \dot{\theta}}{2E_o^{\frac{1}{2}}(\psi(\dot{\theta}))^{3/2}} \qquad (64)$$

$$h = 2\left(\frac{E}{\psi}\right)^{\frac{1}{2}} \delta + \left(\frac{E}{\psi}\right)^{\frac{1}{2}} \left(\dot{\theta}^2 - \frac{D\dot{\theta} + F}{E}\right)\delta + \left(\frac{\Delta}{E}\right)^{\frac{1}{2}} \gamma\Bigg)\frac{\pi^2}{4\dot{\theta}^2} \qquad (65)$$

where $0 < \delta < \infty, -\infty < \gamma < \infty$ and $0 < \dot{\theta} < \infty$.

Equation (64) has been factored into the form

$$f(\delta, \gamma, \dot{\theta}) = f_1(\delta) \cdot f_2(\gamma) \cdot f_3(\theta) \qquad (66)$$

and therefore δ, γ and $\dot{\theta}$ are independent (a surprising result indeed). The wave height is a complicated function of δ, γ, and $\dot{\theta}$. In principle, it ought to be possible to find an appropriate transformation of variables and appropriate ranges of integration to obtain the pdf of f(h), but so far, little success has resulted from this line of investigation.

Instead, for a given set of spectral moments, E_o, E_1, E_2, E_3, E_4, (or m_0, m_1, m_2, m_3, m_4, in other notation) the fact that δ, γ and $\dot{\theta}$ are independent can be used and a random sample of values of h can be generated and compared with experimental data.

The procedure is to select at random triplets of numbers, one from a unit normal distribution for γ and the other two from the rectangular distribution over zero to one. The cumulative density functions for δ is given by

$$P(\delta) = \frac{2}{\sqrt{2\pi}} \left(\int_o^{\delta} e^{-x^2/2}\, dx - \delta\, e^{-\delta^2/2}\right) \qquad (67)$$

where $0 < P(\delta) < 1$ is a number from the 0 to 1 rectangular pdf.

The cumulative density function for $\dot{\theta}$ is given by

$$P(\dot{\theta}) = \frac{E_o^{\frac{1}{2}}}{(E_o E_2)^{\frac{1}{2}} + E_1} \left(E_2^{\frac{1}{2}} - \frac{E_2 - E_1\dot{\theta}}{(E_2 - 2E_1\dot{\theta} + E_o\dot{\theta}^2)^{\frac{1}{2}}}\right) \qquad (68)$$

again where $P(\dot{\theta})$ is a number from the 0 to 1 rectangular pdf.

In finding the cdf, it was necessary to determine a constant K so as to renormalize the pdf of $\dot{\theta}$. It is given by (69).

$$K = \frac{(E_2\ E_o)^{\frac{1}{2}} + E_1}{2\ E_o} \tag{69}$$

To find δ and $\dot{\theta}$, a simple table of $P(\delta)$ versus δ and $P(\dot{\theta})$ versus $\dot{\theta}$ can be constructed, and given $P(\delta)$ and $P(\theta)$ the value of δ and $\dot{\theta}$ can be found.

The randomly chosen values of γ, δ and $\dot{\theta}$ then yield a random value for the wave height h. A sample of, say, 500 values of h ought to yield a fairly reliable cumulative density for h.

Equations (56) and (57) provide some insight of what might be expected. If $\ddot{R}$ were very small (56) would reduce to the Rayleigh pdf. Small values of $\dot{\theta}$, which corresponds to longer than average periods, and higher waves, will, produce frequent negative values of $\ddot{R}$ and thus higher waves may be less frequent than the Rayleigh predicts. Conversely large values of $\dot{\theta}$, which correspond to short periods and low waves will produce positive values of $\ddot{R}$ and hence low waves will be less frequent than the Rayleigh predicts.

The symbols ψ, D, E, F and Δ above are defined as follows:

$$\psi = E_2 - 2E_1\ \dot{\theta} + E_o\ \dot{\theta}^2 \tag{70}$$

$$D = E_o\ E_3 - E_1\ E_2 \tag{71}$$

$$E = E_o\ E_2 - E_1^{\ 2} \tag{72}$$

$$F = E_2^2 - E_1\ E_3 \tag{73}$$

$$\Delta = \begin{vmatrix} E_o & E_1 & -E_2 \\ E_1 & E_2 & -E_3 \\ -E_2 & -E_3 & E_4 \end{vmatrix} \tag{74}$$

The difficulties with theories that depend on the higher moments of wave spectra lie in the increased sampling variability of the moments because the moment spectra are more nearly white noise and in the contamination of the high frequencies in the spectral estimates by digitization and round off errors

in the original wave data.

The definition of the spectral width parameter used by Spring (1978) is (75).

$$\varepsilon^2 = \frac{E_o E_4 - E_2^2}{E_o E_4} \tag{75}$$

in our notation. To avoid the use of spectral moments, Spring (1978) used

$$\varepsilon^2 = 1 - \left(\frac{T_c}{T_z}\right)^2 \tag{76}$$

where ε is the spectral width parameter given in Tables 1, 2 and 3. T_c is the average time interval between crests and T_z is the average time interval between zero up crosses.

For data tabulated in the form of pairs of wave heights and wave "periods", T_z, can be found by averaging the periods and tabulated values of ε as in Table 1 then yield T_c. The reciprocals of these are $\bar{f}_2$ and $\bar{f}_4$ as in the definitions of (77) and (78).

$$E_o \bar{f}_2^2 = \int_o^\infty f^2 S(f)df \tag{77}$$

$$E_o \bar{f}_4^4 = \int_o^\infty f^4 S(f)df \tag{78}$$

and thus $\bar{f}_4 = (1 - \varepsilon^2)^{-\frac{1}{2}} \bar{f}_2$ (79)

To Monte Carlo equations (65) and (66) also requires $\bar{f}_1$ and $\bar{f}_3$ defined by

$$E_o \bar{f}_1 = \int_o^\infty f S(f)df \tag{80}$$

$$E_o \bar{f}_3^3 = \int_o^\infty f^3 S(f)df \tag{81}$$

It can be shown that $\bar{f}_1 < \bar{f}_2 < \bar{f}_3 < \bar{f}_4$, and that (70) to (74) are always positive. Reasonable guesses for $\bar{f}_1$ and $\bar{f}_3$, plus a nondimensionalization of h were then used to generate sample cdf's by a Monte Carlo simulation.

Departures from the Rayleigh were large and not very well related to the actual samples that were used from parts of the total record.

If there is any particular virtue to this theory, it will need to be tested against data samples of sufficient quality to make the moments reliable and that are long enough in duration and stationary (statistically) so that a reliable comparison can be made. The data from Camille lacks stationarity and have other strange properties, as will be shown later.

An Alternate Approach to the Same Problem. Tayfun (1981a) solved a problem for the distribution of crest to trough wave heights in a completely different way for a linear Gaussian model. The pdf requires only the first and second moments of the spectrum, but the spectrum must be narrow band. In his notation,

$$\mu_j = \int_0^\infty \omega^j \, S(\omega) d\omega \tag{82}$$

and the required spectral parameter is

$$\nu^2 = \frac{\mu_2}{\omega_o \omega_1^2} - 1 \tag{83}$$

Where $\omega_o = \mu_1/\mu_o$ (84)

so that ν^2 can be rewritten as

$$\nu^2 = \frac{\mu_2 \, \mu_o - \mu_1^2}{\mu_1^2} \tag{85}$$

It is also required that $\nu^2 << 1$.

The amplitude of the waves is also scaled somewhat differently in this work. If, for example, $\eta(t)$ is scaled by $\xi = \eta(t)/(2\mu_o)^{1/2}$, the Rayleigh pdf takes the normalized form

$$f(x) = 2x \, e^{-x^2} \qquad \text{for } x > 0 \tag{86}$$

Two points on the envelope of the wave time history, separated by the time interval τ, where the envelopes function is found by finding the time history of the quadrature function of η say $\hat{\eta}$ and forming (87),

$$x(t) = (\eta^2 + \hat{\eta}^2)^{\frac{1}{2}} \tag{87}$$

can be shown (probably going back to Rice (1944)), as in Middleton (1960), to have the joint pdf given by (88) with τ as a parameter where x_1 and x_2 are points on the normalized envelope τ units of time apart.

$$f(x_1, x_2; \tau) = \frac{4x_1x_2}{1 - r^2} I_o \left(\frac{2x_1 x_2 r}{1 - r^2}\right) \exp\left(-\frac{x_1^2 + x_2^2}{1 - r^2}\right)$$

$$\text{for } x_1, x_2 \geq 0 \tag{88}$$

where $I_o(\cdot)$ is the zero order modified Bessel function of the first kind.

In (88), r is analogous to a correlation coefficient. It is defined by

$$r(\tau) = (\rho_1^2 + \rho_2^2)^{\frac{1}{2}} \tag{89}$$

where

$$\rho_1(\tau) = \mu_o^{-1} \int_o^\infty S(\omega)(\cos(\omega - \omega_o)\tau)d\omega \tag{90}$$

and

$$\rho_2(\tau) = \mu_o^{-1} \int_o^\infty S(\omega)(\sin(\omega - \omega_o)\tau)d\omega \tag{91}$$

According to Tayfun (1981a), with small changes in his wording, and some deletions;-

The narrow-band condition $\nu^2 << 1$ suggests that the underlying spectrum, $S(\omega)$, is centered sharply around $\omega = \omega_o$, and behaves as a pseudo-delta-function. Correspondingly, each realization of η has the distinctive form of slowly modulated oscillations with an apparent mean frequency, ω_o.

When the spectrum is wide-band so that the condition $\nu^2 << 1$ is not satisfied strictly, the rate of change of the envelope could be rather large. In this case, realizations of η can no longer be viewed as amplitude-modulated oscillations with an apparent mean frequency, ω_o. Consequently, the physical interpretation of the envelope as an amplitude is somewhat lost. Nonetheless, the formalism provided remains valid, and a local wave height can still be defined as H = 2A. Noting that $H_{rms} = 2A_{rms}$, it then follows that the scaled height $\xi = H/H_{rms}$ has the invariant probability density depicted by the Rayleigh form of equation (9), regardless of whether the condition $\nu^2 << 1$ is satisfied or not. -

If x_1 corresponded to a normalized crest at time, t, and a normalized trough followed at t + T/2, a half cycle later then $x_1 + x_2$ would be the normalized crest to trough wave heights.

The general case for which $\tau \neq T/2$ is discussed as related to the wave spectrum through the previous four equations, and involves an integration over the pdf of the distribution of wave periods (halved). This is not attempted and further assumptions are needed. These are obtained by expanding the trigonometric terms in (90) and (91) as Taylor series which gives a simplified expression for $r(\tau)$ in the form of equation (92).

$$r(\tau) \cong \rho(\tau) \cong 1 - (\omega_o \tau \nu)^2/2 \tag{92}$$

The approximation essentially reduces to the assumption that each wave has the same half period given by $\tau = \pi/\omega_o$, and then those values of x_1 and x_2 that add to 2x are found by a convolution by defining

$$x_1 = 2x - u \tag{93}$$

$$x_2 = u \tag{94}$$

and evaluating the integral in (95)

$$f(x) = 2 \int_o^{2x} f(2x - u, u; \frac{\pi}{\omega_o})du \tag{95}$$

This reduces to the computational problem of numerically evaluating (95), given (92) with $\tau = \pi/\omega_o$ since the pdf has been normalized to degenerate to the Rayleigh form for $\nu = 0$. Two examples were computed, one for $\nu = 0.1$ and the other for $\nu = 0.2$. Fig. 17 shows the result for $\nu = 0.2$ compared to the Rayleigh pdf. The points for $\nu = 0.1$ are about 0.01 higher than the Rayleigh pdf at the peak, and $\nu = 0.2$ pushes the approximations that were made about as far as possible.

These results verify the guesses that were made in the study by Pierson and Salfi (1982) and obtain them in a much simpler way, but the results would not apply too well for hurricane waves. The results also imply some problems in verification to be pointed out later since successive waves would, of necessity, have to be highly correlated.

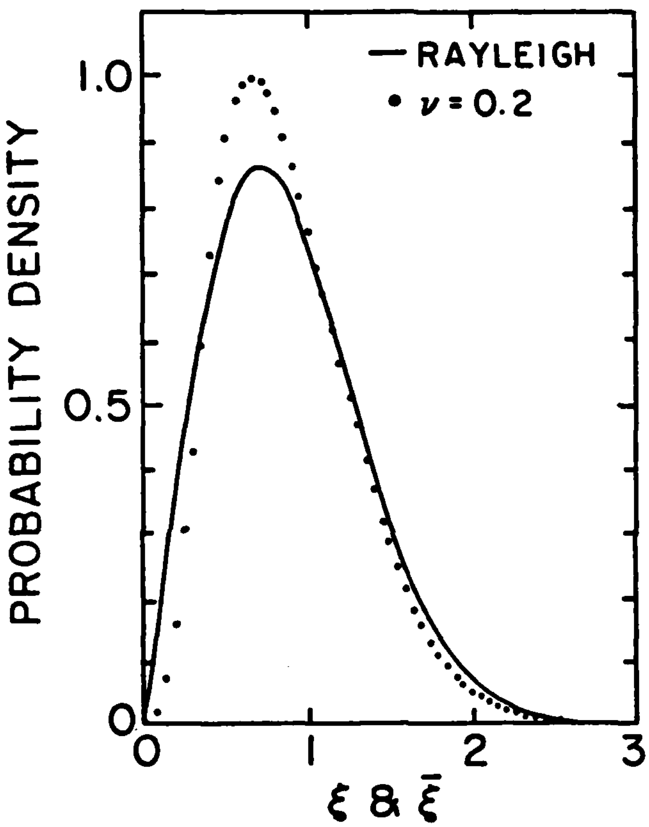


FIGURE 17 Generalization of the Crest to Trough Wave Height Probability Density Function for a Narrow Band Spectrum (Redrafted from Tayfun (1981a), with the Points for $\nu = 0.1$ Omitted).

<u>Effects of Breaking Waves</u>. If waves were Gaussian, and if it would still be possible to consider the effects of breaking within the constraints of a linear model, or perhaps a second order model because the second harmonic could cancel for crest to trough heights, then one would expect some of the higher waves in a high enough seaway to be limited in height by breaking. The higher waves predicted by the Rayleigh pdf would each be some fraction lower had they broken prior to passing some point where they are measured as a function of time.

Counter effects, as the waves approach the measurement point, are the steepening just before breaking with the formation of a sharp angular crest with a corner flow as described previously. Harmonic analysis is probably the wrong way to attempt to describe the waves since a large number of bound harmonics would be needed. The crest to trough height of a wave about to break would be greater by some unknown fraction than predicted by a linear or second order model.

Tayfun ((1981b),(1983)) has treated the limitations of wave height by breaking in a linear Gaussian model (with reasons why it probably still applies at second order). The mathematics of the derivation is contained in the above reference.

With β defined as

$$\beta = k_e(2\ \mu_o)^{\frac{1}{2}} \tag{96}$$

to represent a measure of wave steepness where

$$k_e = \omega_e^2/g \tag{97}$$

for $\omega_e = (1 + \nu^2)^{\frac{1}{2}}\ \omega_o$ (see equation (83)), (98)
and N defined as

$$N = (\pi/7\beta)^2 \tag{99}$$

for a normalized wave height given by

$$\xi = \frac{h}{2(2\mu_o)^{\frac{1}{2}}} \tag{100}$$

the modified pdf is given by

$$f(x) = xN \left| 1 - \frac{4}{\pi} \cos^{-1}\left(\frac{N^{\frac{1}{2}}}{\xi} \right) \quad U\left(1 - \frac{N^{\frac{1}{2}}}{\xi} \right) \right|$$

$$\cdot \int_{o}^{\infty} u \quad J_o^N \,(u) \; J_o(u\xi N^{\frac{1}{2}})du \tag{101}$$

where U is the Heaviside unit step function, $J_o(\cdot)$ is the zero order Bessel function, and $0 \leq x \leq (2N)^{\frac{1}{2}}$.

Figure 18 from Tayfun (1981) shows both a different way to plot the data and the behavior of a mapped inverse cdf as N is varied. Values for N = 5, 10 and ∞ are shown. On the original, N = 15, 30 and 60 (which falls halfway between 10 and ∞) are also shown.

For this type of normalization, note that the normalized cdf for $N = \infty$ is given by

$$F(\xi) = 1 - \exp\,(-(\xi)^2) \tag{102}$$

which can be transformed to

$$\xi = (-\ell n\ (1 - F(\xi)))^{\frac{1}{2}} \tag{103}$$

so that the log of $(1 - F(\xi))$ can be graphed against ξ starting from $\xi = 0$ where $F(\xi) = 0$ and ℓn 0 has an antilog of one. The curve that results is the probability that ξ exceeds the value plotted on the vertical axis. A sample point at 10^{-3} requires 1000 independent value from the sample for verification.

An example in Tayfun (1983) of the pdf plotted in the usual way shows slight deficits compared to the Rayleigh for low values of the scaled height, excesses past the maximum of the pdf and, as expected, deficits past a scaled height of about 2. The differences between this result and the result for a narrow band spectrum without breaking is that the pdf for the narrow band process peaks near the maximum of the Rayleigh whereas the breaking wave model corrects the pdf with higher values past the peak. Both pdf's are lower than the Rayleigh for larger values of the scaled height.

For both of these models, the significant wave height and the average of

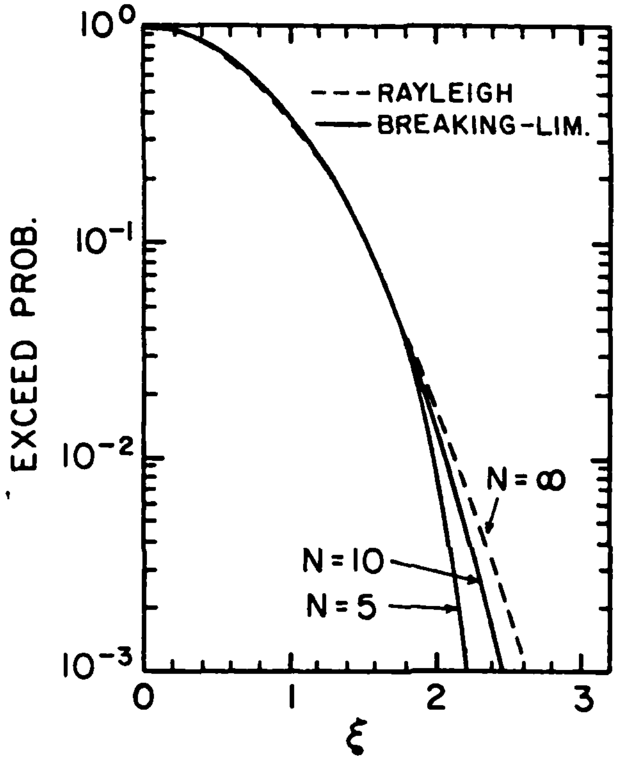


FIGURE 18 Modified Wave Height Exceedence Probabilities from Tayfun (1981b). (Curves for N = 15, 30, and 60 Have Been Omitted).

the one tenth highest waves would be functions of three parameters from the spectrum, i.e. μ_o, μ_1 and μ_2, which in turn can be used to compute all of the other parameters that arise in the derivation. The spectra would have to be quite well known, either theoretically or as estimated from a high quality time history. Given a sample of crest to trough wave heights, and the moments, whether or not the presently available tests are good enough to distinguish between these models and the Rayleigh would be a matter of interest.

Thornton and Guza (1983), in a study of the pdf's of waves as they travel through the breaker zone showed that the Rayleigh distribution fitted to the root mean square height described the significant height and the height of the one tenth highest with average errors of -0.2 % and -1.8%, respectively, both before the breaker zone and after reforming after breaking with some energy loss. A model based on a shoaling coefficient and a single free parameter explained the data within ± 9%.

Other models to account for wave breaking in shoaling water, as a review of the results of five different authors, were also given, but none of them would have given a Rayleigh pdf in the shallower water. They write, after reviewing these various models, and the theories that have been developed to modify the wave height distribution in deep water, that "These studies seek to explain deviations from a Rayleigh distribution, but the relevant point here is that wave heights appear to be nearly Rayleigh under a much wider range of conditions than the strict assumptions of a narrow band Gaussian (linear) process would imply".

Definition and Consequences Thereof. If the random variables X_1, X_2, $X_3 \ldots X_n$ have a joint density

$f_{(x_1, \ldots x_n)}(x_1, x_2, x_3 \ldots x_n)$ that factors into the form

$f(x_1)\, f(x_2) \ldots . f(x_n)$ where $f(\cdot)$ is the same pdf for each X_i, then X_1, $X_2 \ldots X_n$ is defined to be a *random sample* of size n from a population with a pdf, $f(\cdot)$. See, for example, Mood, et al. (1963). The problem is that successive crest to trough ocean wave heights may not meet the requirement that they form an independent random sample.

In the spectral domain, the difficulty is overcome and successive spectral estimates as the frequency varies obtained via an FFT are independent. It could then be shown that 1024 points equally spaced from a time history would be the rough equivalent of 150 values from a random sample for the waves studied by Donelan and Pierson (1983). The more narrow band the spectrum the fewer the effective number of independent points for a given time history.

For the estimated spectrum obtained from measurement of a wind generated sea, it is possible to compute the covariance function as in, say,

$$\mathcal{R}(\tau) = \int_0^\infty S(\omega) \cos \omega\tau \, d\omega \qquad (104)$$

where $\mathcal{R}(0) = E_o = m_o = \mu_o$

the function, $\mathcal{R}(\tau)$, is at first positive, then goes negative by perhaps 20 to 30% of $\mathcal{R}(o)$, and then positive again. Successive crest to trough wave heights may consequently be correlated, and if the X_i represent wave height the joint pdf of X_1, $X_2 \ldots X_n$ may not be factorable into the form required for an independent random sample.

Most of the wave theories described above are either for periodic waves (2 degrees of freedom for a linear theory), or slowly varying functions of x and t, or for waves with a narrow band spectrum. Wave height samples under these assumptions do not qualify as random samples. One of the problems to be addressed in the following is the effective numbers of

independent wave heights in a sample of say, N, successive wave heights from an ocean wave time history.

There are two tests that are routinely applied to determine whether or not a given random sample could have been obtained from a population with a certain pdf. One is the Chi Square goodness-of-fit test based on a sample histogram. The other is the Kolmogorov-Smirnov goodness-of-fit test based on a sample cdf. The latter is not applicable to this particular area because all parameters of the pdf need to be known a priori.

The usual Chi Square test has two hidden traps in it. One is the not very well specified requirement to pool the tails of the sample histogram before Chi Square is found. Some light will be shed on this rather vague rule. The other is that the test gives too high a value for Chi Square if the sample is not an independent random sample.

<u>The Chi Square Test</u>. Given a theoretical pdf of the form, $f(x_i; p_1, p_2, \ldots p_q)$ that depends on q parameters, that in general have to be estimated from the sample as say $\hat{p}_1, \hat{p}_2, \ldots \hat{p}_q$, and an independent random sample, $X_1, X_2, \ldots X_n$, as defined above, it is possible to divide the range of X into some k + 1 mutually exclusive sets, say, those for the X's such that

$$-\infty < X_i \leq X_a,$$

$$X_a < X_i \leq X_a + \Delta X$$

$$X_a + \Delta X < X_i \leq X_a + 2\Delta X$$

and so on to

$$X_a + k\Delta X < X_i < \infty .$$

The number of X_i that fall in each set can be counted to find N_j as j goes from 1 to k + 1. The probability that a given X_i will fall in one of the ranges defined above is the integral of the theoretical pdf over the range defined above, say, p_j^o and the expected number, if the hypothesis that the sample came from the assumed pdf is made, is given by np_j where n is the total sample size.

The quality Q_k^o can then be computed as equation (105).

$$Q_k^o = \sum_{j=1}^{k+1} \frac{(N_j - n\, p_j^o)^2}{n p_j^o} \tag{105}$$

Under these circumstances Q_k^o has a limiting distribution, as n approaches infinity, of Chi Square with k - q degrees of freedom where q is the number of parameters determined from the sample. Just how large n ought to be to use the test, which is a large sample test, is open and if the q parameters are estimated efficiently some of the lost degrees of freedom can be recouped. (Mood, et al. (1963) pg 443).

Suppose now that the n values of X_i are not independent and that the sample is somehow mysteriously disguised such that the true sample size is $n_T = \alpha n$ where $0 < \alpha < 1$. To pick an example, suppose the 200 successive wave heights in a sample from Camille are effectively the equivalent of only 100 independent sample values.

A corrected value of Q_k^o would be given by

$$Q_k^o = \sum_{j=1}^{k+1} \frac{(\alpha N_j - \alpha n p_j^o)^2}{\alpha\, n\, p_j^o} = \alpha\, Q_k^o \qquad (106)$$

and the Chi Square value to be used to test the hypothesis that the sample comes from $f(X, P_1, P_2, \ldots, Pq)$ would be halved. For, say, $k + 1 = 15$ and $q = 2$, the appropriate Chi Square has 12 degrees of freedom and if Q_k^o were to exceed 21, the hypothesis would be rejected at the 5% level. Actually the value is closer to 10.5, which could easily happen more than half the time. For the study of successive crest to trough wave heights in a time history, something equivalent to the effective number of independent samples value, as needed for spectral estimation, is needed before goodness-of-fit tests can be made.

MARKOV CHAINS

Introduction. If successive crest to trough wave heights were independent, then many of the assumptions in the previous derivations would not be very good. If they were independent, the waves might have very strange properties compared to what is believed to be known about them. One of the simplest ways to model a process that does not have the properties required for an independent random sample is to study Markov chains. This process simply assumes that the probability of the next event in a sequence depends only on the present state of the process.

Each of the fifty five samples of 200 successive waves was treated as if it were a ten state Markov process. The 200 waves were ordered in ascending order from the lowest to the highest. State one for a Markov process was defined to be the 20 lowest waves; state 2 the next 20 and so on to state 10 as the highest 20.

With the waves in their original order, the first wave in the sequence of 200 would be in one of the ten states. The next wave would also be in one of the ten states. The transition probability can then be estimated by counting the number of times one of the waves in state 1 is followed by a wave in states 1 to 10, counting the number of times one of the waves in state 2 is followed by a wave in states 1 to 10 and so on to state 10 for the initial state. To keep things simple, it is necessary to look ahead to wave 201, the first wave of the next sample.

As examples, the transition counts for records 6 to 10 are shown in Tables 4-6 to 4-10. Each row sums to twenty. Since there are ten possible transition states, if wave heights were completely random, there would be on the average two counts in each element of each row. The chance of all twos in every element of the matrix is small so that 200 wave samples are not large enough to provide useful estimates.

In these five tables the counts scatter from zero to six (the sixes are in 4-6, state 10 to state 4 and in 4-10, state 8 to state 9). The individual values are not much different from a sample from a binomial discret density function with $p = 0.1$ and $n = 20$.

Each table of counts also shows the length of time in minutes for the waves in the sample, the highest wave in the sample and the "period" of that wave. These values were summarized at the start by Fig. 4 and Table 4.

With 55 successive wave samples for a total of 11,000 waves during Camille, the ordering procedure for 200 wave sets has taken out most of the effect of trends in wave height. The arrays of sample counts for sets of five records 1 to 5, 6 to 10, 11 to 15, and so on to 51 to 55 were pooled to form a count matrix for 1000 waves yielding eleven arrays for 1,000 waves each.

Table 4-C-2-1 shows the count array for the second 1,000 wave combined sample. Each row totals to 100, and for completely random waves each element of each row should be 10. The difference between what was actually counted and 10 is shown in the second array.

There are no zeros in the array. A wave in any state (10% height range) can be followed by a wave in any of the ten states. The lowest values in the array are 5's and the highest values are 18's. A binomial distribution with p = 0.10, n = 100 would have an expected value of 10 and twice the standard deviation would be 6. Since the distribution is skewed, a range from 5 to 18 is not unusual.

When converted to the transition probability matrix, the result is shown in Table 4-C-2-2. Any initial state, can change to any other initial state. A wave in the lowest 10% can be followed by a wave from the highest 10% with an estimated sample probability of 0.12. Each probability for a completely unpredictable sequence of heights ought to be 0.10. It is clear that for this 1000 wave set, the prediction of the height of the wave that follows a particular wave, given its height as a number from one to ten is practically a random choice at this degree of resolution.

If successive wave heights could be treated as a Markov process, the square of the transition matrix would give the probabilities for the states of the second wave to follow a wave in a known state. The square of the matrix in 4-C-2-2 is shown in 4-C-2-3. The values differ from 0.10 only in the third decimal place except for state 6. Under these assumptions,

knowing the height of a passing wave provides very little knowledge of how high the second wave to follow it will be. The cube of the matrix in the next table differs from 0.10 by at most three parts in 1000.

The actual two step count transition probability matrix, and its square are given in Tables 4-C22-1, 4-C22-2 and 4-C22-3. The element of randomness dominates. The counts scatter from 5 to 18 again, whereas Markov theory based on the square of the one step matrix would predict all elements to be much closer to 10. There is little, or no, apparent relationship between the one step counts and the two step counts.

The square of the actual two step matrix gives an idea of the chance of predicting the fourth wave to follow a wave of known state. Pick a number at random from one to ten.

Tables 5-51 to 5-56, 5-C-11-1, and so on, provide similar tables for the last five samples of 200 waves and the result of combining the five samples into one 1000 wave sample. The contrast between 4-C-2-1 and 5-C-11-1 is striking. The values near the main diagonal tend to be above ten and those in the upper right and lower left tend to be below ten. Although any state could go into any other state, a 10% lowest wave goes to the 10% highest only twice. The counts for the transition of any wave in states one to four to states eight to ten are all under ten. The transition from state ten to state ten was two and a half times greater than the chance value with a sample probability of 0.25.

The square of the transition matrix indicates some residual, but weak, capability to predict the second wave to follow a wave in a known state. Some of the values differ from 0.10 in the second decimal place, and the signs of the differences are negative in the upper right and the lower left corners and positive in the upper left and lower right corners.

The actual two step counts, however, do not verify the square of the one step count matrix. Counts above ten and below ten scatter at random throughout the array. The square of the two step matrix to predict the fourth wave differs from 0.1 only in the third decimal place.

For completeness, the three step count array and the corresponding matrices are given in Tables 5-C-113-1, 5-C113-2 and 5-C-113-3. The counts

are essentially random numbers with no discernable pattern. The square of the matrix, which might be the prediction of the sixth wave to follow a wave of known height, can be judged by inspection to be of no use.

Tables 6-1, 6-3, 6-4, to 6-10 give the 1000 wave arrays for the nine ten state Markov processes not given in the previous tables. The number after the dash corresponds to the number of the 1000 wave sample. There are 11, and the missing 6-2 and 6-11 are given in 4-C-2-1 and 5-C-11-1. The trend for the high counts to concentrate along the main diagonal toward the end of the total record can be seen in the data.

TABLE 4-6 One Step Transition Counts and Sample Markov Transition Matrix for Record 6.

REC	1 STEP COUNTS 200 WAVES											LEN	HMAX	TMAX
6		1	2	3	4	5	6	7	8	9	10	16.72	5.84	4.63
1	*	3	1	2	2	4	0	1	0	4	3	* 20		
2	*	2	1	2	0	0	4	2	5	2	2	* 20		
3	*	1	5	0	3	2	1	2	1	3	2	* 20		
4	*	2	3	2	0	5	1	3	3	1	0	* 20		
5	*	1	3	2	3	0	2	3	1	3	2	* 20		
6	*	2	2	2	0	3	1	2	5	0	3	* 20		
7	*	2	3	3	2	1	1	2	2	3	1	* 20		
8	*	3	2	3	2	3	2	0	1	0	4	* 20		
9	*	1	0	3	2	1	6	3	1	2	1	* 20		
10	*	3	0	0	6	2	2	2	1	2	2	* 20		
		20	20	19	20	21	20	20	20	20	20	* 200		

1 STEP PROBABILITY

		1	2	3	4	5	6	7	8	9	10
1	*	0.15	0.05	0.10	0.10	0.20	0.00	0.05	0.00	0.20	0.15
2	*	0.10	0.05	0.10	0.00	0.00	0.20	0.10	0.25	0.10	0.10
3	*	0.05	0.25	0.00	0.15	0.10	0.05	0.10	0.05	0.15	0.10
4	*	0.10	0.15	0.10	0.00	0.25	0.05	0.15	0.15	0.05	0.00
5	*	0.05	0.15	0.10	0.15	0.00	0.10	0.15	0.05	0.15	0.10
6	*	0.10	0.10	0.10	0.00	0.15	0.05	0.10	0.25	0.00	0.15
7	*	0.10	0.15	0.15	0.10	0.05	0.05	0.10	0.10	0.15	0.05
8	*	0.15	0.10	0.15	0.10	0.15	0.10	0.00	0.05	0.00	0.20
9	*	0.05	0.00	0.15	0.10	0.05	0.30	0.15	0.05	0.10	0.05
10	*	0.15	0.00	0.00	0.30	0.10	0.10	0.10	0.05	0.10	0.10

TABLE 4-7 One Step Transition Counts and Sample Markov Transition Matrix for Record 7.

REC	1 STEP COUNTS 200 WAVES										LEN	HMAX	TMAX
7	1	2	3	4	5	6	7	8	9	10	16.65	5.89	4.32
1 *	3	1	4	2	1	3	0	2	2	2 *	20		
2 *	0	3	2	5	1	3	2	2	1	1 *	20		
3 *	3	3	3	3	1	1	1	3	1	1 *	20		
4 *	2	2	1	3	3	0	1	1	4	3 *	20		
5 *	1	1	0	2	2	4	5	1	2	2 *	20		
6 *	0	3	4	1	2	1	2	4	2	1 *	20		
7 *	1	0	1	2	3	3	2	4	1	3 *	20		
8 *	2	1	1	1	3	1	1	1	5	4 *	20		
9 *	5	4	2	1	1	2	2	1	2	0 *	20		
10 *	3	2	2	0	2	3	4	1	0	3 *	20		
	20	20	20	20	19	21	20	20	20	20 *	200		

1 STEP PROBABILITY

	1	2	3	4	5	6	7	8	9	10
1 *	0.15	0.05	0.20	0.10	0.05	0.15	0.00	0.10	0.10	0.10
2 *	0.00	0.15	0.10	0.25	0.05	0.15	0.10	0.10	0.05	0.05
3 *	0.15	0.15	0.15	0.15	0.05	0.05	0.05	0.15	0.05	0.05
4 *	0.10	0.10	0.05	0.15	0.15	0.00	0.05	0.05	0.20	0.15
5 *	0.05	0.05	0.00	0.10	0.10	0.20	0.25	0.05	0.10	0.10
6 *	0.00	0.15	0.20	0.05	0.10	0.05	0.10	0.20	0.10	0.05
7 *	0.05	0.00	0.05	0.10	0.15	0.15	0.10	0.20	0.05	0.15
8 *	0.10	0.05	0.05	0.05	0.15	0.05	0.05	0.05	0.25	0.20
9 *	0.25	0.20	0.10	0.05	0.05	0.10	0.10	0.05	0.10	0.00
10 *	0.15	0.10	0.10	0.00	0.10	0.15	0.20	0.05	0.00	0.15

TABLE 4-8 One Step Transition Counts and Sample Markov Transition Matrix for Record 8.

REC		1 STEP COUNTS 200 WAVES										LEN	HMAX	TMAX
8		1	2	3	4	5	6	7	8	9	10	17.91	6.07	5.73
1	*	2	1	3	1	3	3	1	3	2	1	* 20		
2	*	3	4	3	2	2	2	1	1	2	0	* 20		
3	*	3	2	3	3	0	0	3	0	3	3	* 20		
4	*	3	1	1	1	2	4	0	3	1	4	* 20		
5	*	1	2	4	2	2	3	3	2	1	0	* 20		
6	*	4	2	1	1	1	2	0	2	4	3	* 20		
7	*	1	5	2	0	3	0	4	2	2	1	* 20		
8	*	2	1	1	7	2	0	1	2	2	2	* 20		
9	*	1	1	2	0	0	5	5	3	0	3	* 20		
10	*	0	1	0	3	4	1	3	2	3	3	* 20		
		20	20	20	20	19	20	21	20	20	20	* 200		

1 STEP PROBABILITY

		1	2	3	4	5	6	7	8	9	10
1	*	0.10	0.05	0.15	0.05	0.15	0.15	0.05	0.15	0.10	0.05
2	*	0.15	0.20	0.15	0.10	0.10	0.10	0.05	0.05	0.10	0.00
3	*	0.15	0.10	0.15	0.15	0.00	0.00	0.15	0.00	0.15	0.15
4	*	0.15	0.05	0.05	0.05	0.10	0.20	0.00	0.15	0.05	0.20
5	*	0.05	0.10	0.20	0.10	0.10	0.15	0.15	0.10	0.05	0.00
6	*	0.20	0.10	0.05	0.05	0.05	0.10	0.00	0.10	0.20	0.15
7	*	0.05	0.25	0.10	0.00	0.15	0.00	0.20	0.10	0.10	0.05
8	*	0.10	0.05	0.05	0.35	0.10	0.00	0.05	0.10	0.10	0.10
9	*	0.05	0.05	0.10	0.00	0.00	0.25	0.25	0.15	0.00	0.15
10	*	0.00	0.05	0.00	0.15	0.20	0.05	0.15	0.10	0.15	0.15

TABLE 4-9 One Step Transition Counts and Sample Markov Transition Matrix for Record 9.

REC	1 STEP COUNTS 200 WAVES											LEN	HMAX	TMAX
9	1	2	3	4	5	6	7	8	9	10		17.99	6.44	7.31
1	2	1	3	4	0	2	0	2	2	4	*	20		
2	2	4	2	0	1	1	2	6	1	1	*	20		
3	2	2	2	1	2	2	3	1	2	3	*	20		
4	0	1	1	2	6	3	1	0	3	3	*	20		
5	2	1	4	3	3	2	1	0	2	2	*	20		
6	2	1	4	3	4	3	0	1	0	2	*	20		
7	3	3	1	2	2	2	3	2	1	1	*	20		
8	5	3	0	1	0	1	2	2	5	1	*	20		
9	1	2	1	3	0	4	5	1	1	2	*	20		
10	1	2	2	1	2	1	3	4	3	1	*	20		
	20	20	20	20	20	21	20	19	20	20	*	200		

1 STEP PROBABILITY

	1	2	3	4	5	6	7	8	9	10
1 *	0.10	0.05	0.15	0.20	0.00	0.10	0.00	0.10	0.10	0.20
2 *	0.10	0.20	0.10	0.00	0.05	0.05	0.10	0.30	0.05	0.05
3 *	0.10	0.10	0.10	0.05	0.10	0.10	0.15	0.05	0.10	0.15
4 *	0.00	0.05	0.05	0.10	0.30	0.15	0.05	0.00	0.15	0.15
5 *	0.10	0.05	0.20	0.15	0.15	0.10	0.05	0.00	0.10	0.10
6 *	0.10	0.05	0.20	0.15	0.20	0.15	0.00	0.05	0.00	0.10
7 *	0.15	0.15	0.05	0.10	0.10	0.10	0.15	0.10	0.05	0.05
8 *	0.25	0.15	0.00	0.05	0.00	0.05	0.10	0.10	0.25	0.05
9 *	0.05	0.10	0.05	0.15	0.00	0.20	0.25	0.05	0.05	0.10
10 *	0.05	0.10	0.10	0.05	0.10	0.05	0.15	0.20	0.15	0.05

TABLE 4-10 One Step Transition Counts and Sample Markov Transition Matrix for Record 10.

REC 1 STEP COUNTS 200 WAVES LEN HMAX TMAX

REC	1	2	3	4	5	6	7	8	9	10	LEN	HMAX	TMAX
10											18.57	7.67	5.[illegible]3

	1	2	3	4	5	6	7	8	9	10	*	
1 *	2	2	4	0	0	2	2	4	2	2	*	20
2 *	3	3	3	3	1	4	0	1	0	2	*	20
3 *	1	3	0	3	5	2	3	1	1	1	*	20
4 *	2	0	2	2	2	3	1	3	2	3	*	20
5 *	0	1	4	1	0	1	4	3	3	3	*	20
6 *	2	3	3	1	4	2	0	3	2	0	*	20
7 *	2	3	2	0	1	3	1	1	2	5	*	20
8 *	1	1	0	3	2	2	3	2	6	0	*	20
9 *	3	2	1	5	4	0	3	0	1	1	*	20
10 *	4	2	1	2	1	1	3	2	1	3	*	20
	20	20	20	20	20	20	20	20	20	20	*	200

1 STEP PROBABILITY

	1	2	3	4	5	6	7	8	9	10
1 *	0.10	0.10	0.20	0.00	0.00	0.10	0.10	0.20	0.10	0.10
2 *	0.15	0.15	0.15	0.15	0.05	0.20	0.00	0.05	0.00	0.10
3 *	0.05	0.15	0.00	0.15	0.25	0.10	0.15	0.05	0.05	0.05
4 *	0.10	0.00	0.10	0.10	0.10	0.15	0.05	0.15	0.10	0.15
5 *	0.00	0.05	0.20	0.05	0.00	0.05	0.20	0.15	0.15	0.15
6 *	0.10	0.15	0.15	0.05	0.20	0.10	0.00	0.15	0.10	0.00
7 *	0.10	0.15	0.10	0.00	0.05	0.15	0.05	0.05	0.10	0.25
8 *	0.05	0.05	0.00	0.15	0.10	0.10	0.15	0.10	0.30	0.00
9 *	0.15	0.10	0.05	0.25	0.20	0.00	0.15	0.00	0.05	0.05
10 *	0.20	0.10	0.05	0.10	0.05	0.05	0.15	0.10	0.05	0.15

TABLE 4-C-2-1. Summed Transition Counts for Records 6 to 10 and the Difference Between the Values of Ten and the Actual Value.

REC 1 STEP COUNTS 1000 WAVES

REC	1	2	3	4	5	6	7	8	9	10	LEN	HMAX	TMAX
2											87.84	7.67	5.93
1	12	6	16	9	8	10	4	11	12	12	100		
2	10	15	12	10	5	14	7	15	6	6	100		
3	10	15	8	13	10	6	12	6	10	10	100		
4	9	7	7	8	18	11	6	10	11	13	100		
5	5	8	14	11	7	12	16	7	11	9	100		
6	10	11	14	6	14	9	4	15	8	9	100		
7	9	14	9	6	10	9	12	11	9	11	100		
8	13	8	5	14	10	6	7	8	18	11	100		
9	11	9	9	11	6	17	18	6	6	7	100		
10	11	7	5	12	11	8	15	10	9	12	100		
	100	100	99	100	99	102	101	99	120	100	1000		

COUNTS - 10

	1	2	3	4	5	6	7	8	9	10
1	2	-4	6	-1	-2	0	-6	1	2	2
2	0	5	2	0	-5	4	-3	5	-4	-4
3	0	5	-2	3	0	-4	2	-4	0	0
4	-1	-3	-3	-2	8	1	-4	0	1	3
5	-5	-2	4	1	-3	2	6	-3	1	-1
6	0	1	4	-4	4	-1	-6	5	-2	-1
7	-1	4	-1	-4	0	-1	2	1	-1	1
8	3	-2	-5	4	0	-4	-3	-2	8	1
9	1	-1	-1	1	-4	7	8	-4	-4	-3
10	1	-3	-5	2	1	-2	5	0	-1	2

TABLE 4-C-2-2. Transition Probability Matrix for 1000 Waves and the Difference Between it and 0.10 for each Element.

1 STEP PROBABILITY 1000 WAVES

	1	2	3	4	5	6	7	8	9	10
1	0.120	0.060	0.160	0.090	0.080	0.100	0.040	0.110	0.120	0.120
2	0.100	0.150	0.120	0.100	0.050	0.140	0.070	0.150	0.060	0.060
3	0.100	0.150	0.080	0.130	0.100	0.060	0.120	0.060	0.100	0.100
4	0.090	0.070	0.070	0.080	0.180	0.110	0.060	0.100	0.110	0.130
5	0.050	0.080	0.140	0.110	0.070	0.120	0.160	0.070	0.110	0.090
6	0.100	0.110	0.140	0.060	0.140	0.090	0.040	0.150	0.080	0.090
7	0.090	0.140	0.090	0.060	0.100	0.090	0.120	0.110	0.090	0.110
8	0.130	0.080	0.050	0.140	0.100	0.060	0.070	0.080	0.180	0.110
9	0.110	0.090	0.090	0.110	0.060	0.170	0.180	0.060	0.060	0.070
10	0.110	0.070	0.050	0.120	0.110	0.080	0.150	0.100	0.090	0.120

PROB MATRIX - 0.100

	1	2	3	4	5	6	7	8	9	10
1	0.020	-0.040	0.060	-0.010	-0.020	0.000	-0.060	0.010	0.020	0.020
2	0.000	0.050	0.020	0.000	-0.050	0.040	-0.030	0.050	-0.040	-0.040
3	0.000	0.050	-0.020	0.030	0.000	-0.040	0.020	-0.040	0.000	0.000
4	-0.010	-0.030	-0.030	-0.020	0.080	0.010	-0.040	0.000	0.010	0.030
5	-0.050	-0.020	0.040	0.010	-0.030	0.020	0.060	-0.030	0.010	-0.010
6	0.000	0.010	0.040	-0.040	0.040	-0.010	-0.060	0.050	-0.020	-0.010
7	-0.010	0.040	-0.010	-0.040	0.000	-0.010	0.020	0.010	-0.010	0.010
8	0.030	-0.020	-0.050	0.040	0.000	-0.040	-0.030	-0.020	0.080	0.010
9	0.010	-0.010	-0.010	0.010	-0.040	0.070	0.080	-0.040	-0.040	-0.030
10	0.010	-0.030	-0.050	0.020	0.010	-0.020	0.050	0.000	-0.010	0.020

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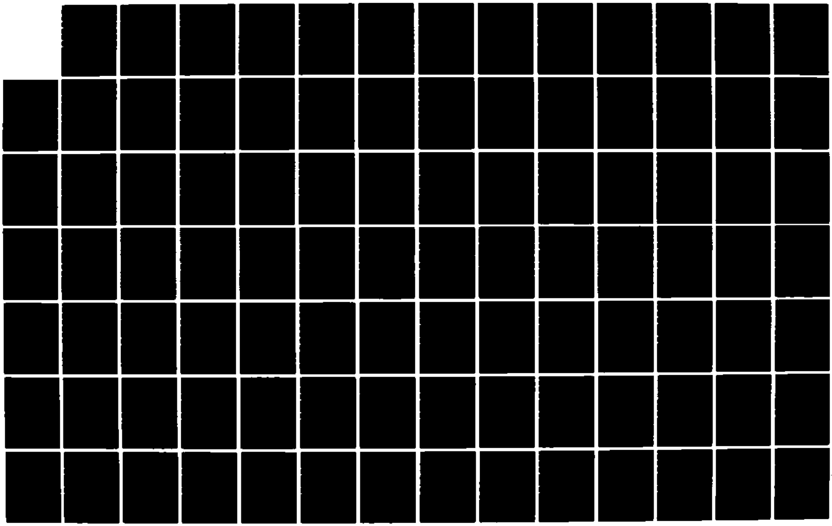

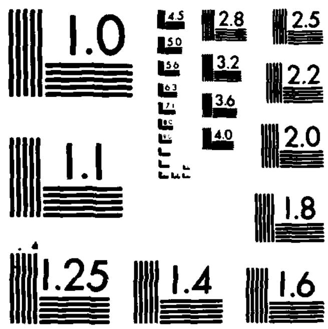

MICROCOPY RESOLUTION TEST CHART
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TABLE 4-C-2-3. The Square of the Matrix in 4-C-2-2 and the Difference Between it and 0.10.

PROB MATRIX SQUARED

	1	2	3	4	5	6	7	8	9	10
1	0.103	0.098	0.097	0.105	0.100	0.099	0.103	0.094	0.102	0.101
2	0.104	0.104	0.099	0.101	0.101	0.098	0.087	0.105	0.103	0.099
3	0.098	0.102	0.100	0.098	0.097	0.107	0.102	0.101	0.096	0.099
4	0.097	0.094	0.101	0.102	0.097	0.104	0.108	0.096	0.101	0.099
5	0.099	0.107	0.095	0.097	0.103	0.100	0.104	0.099	0.096	0.100
6	0.100	0.100	0.100	0.107	0.094	0.099	0.101	0.095	0.105	0.098
7	0.100	0.103	0.099	0.101	0.094	0.102	0.102	0.101	0.099	0.098
8	0.101	0.092	0.099	0.100	0.098	0.111	0.107	0.095	0.098	0.100
9	0.099	0.105	0.104	0.090	0.105	0.099	0.092	0.107	0.097	0.102
10	0.099	0.096	0.098	0.097	0.101	0.102	0.103	0.099	0.102	0.103

PROB MATRIX - 0.100

	1	2	3	4	5	6	7	8	9	10
1	0.003	-0.002	-0.003	0.005	-0.000	-0.001	0.003	-0.006	0.002	0.001
2	0.004	0.004	-0.001	0.001	0.001	-0.002	-0.013	0.005	0.003	-0.001
3	-0.002	0.002	-0.000	-0.002	-0.003	0.007	0.002	0.001	-0.004	-0.001
4	-0.003	-0.006	0.001	0.002	-0.003	0.004	0.008	-0.004	0.001	-0.001
5	-0.001	0.007	-0.005	-0.003	0.003	0.000	0.004	-0.001	-0.004	0.000
6	0.000	-0.000	-0.000	0.007	-0.006	-0.001	0.001	-0.005	0.005	-0.002
7	0.000	0.003	-0.001	0.001	-0.006	0.002	0.002	0.001	-0.001	-0.002
8	0.001	-0.008	-0.001	0.000	-0.002	0.011	0.007	-0.005	-0.002	0.000
9	-0.001	0.005	0.004	-0.010	0.005	-0.001	-0.008	0.007	-0.003	0.002
10	-0.001	-0.004	-0.002	-0.003	0.001	0.002	0.003	-0.001	0.002	0.003

TABLE 4-C-2-4. The Cube of the Matrix in 4-C-2-2 and the corresponding Differences.

PROB MATRIX CUBED

	1	2	3	4	5	6	7	8	9	10
1	0.100	0.100	0.099	0.100	0.099	0.102	0.101	0.099	0.100	0.100
2	0.100	0.099	0.099	0.101	0.099	0.102	0.101	0.099	0.100	0.100
3	0.100	0.100	0.099	0.100	0.099	0.102	0.100	0.100	0.100	0.100
4	0.100	0.100	0.099	0.099	0.100	0.102	0.101	0.099	0.100	0.100
5	0.100	0.100	0.099	0.100	0.099	0.102	0.101	0.100	0.100	0.100
6	0.100	0.100	0.099	0.100	0.099	0.103	0.101	0.099	0.099	0.100
7	0.100	0.100	0.099	0.100	0.099	0.102	0.100	0.100	0.100	0.100
8	0.100	0.100	0.099	0.099	0.100	0.102	0.101	0.099	0.100	0.100
9	0.100	0.100	0.099	0.101	0.098	0.102	0.101	0.099	0.101	0.100
10	0.100	0.100	0.099	0.100	0.099	0.102	0.102	0.099	0.100	0.100

PROB MATRIX - 0.100

	1	2	3	4	5	6	7	8	9	10
1	-0.000	-0.000	-0.001	-0.000	-0.001	0.002	0.001	-0.001	-0.000	0.000
2	0.000	-0.001	-0.001	0.001	-0.001	0.002	0.001	-0.001	0.000	-0.000
3	0.000	0.000	-0.001	-0.000	-0.001	0.002	0.000	-0.000	-0.000	-0.000
4	-0.000	0.000	-0.001	-0.001	-0.000	0.002	0.001	-0.001	-0.000	0.000
5	-0.000	0.000	-0.001	-0.000	-0.001	0.002	0.001	-0.000	-0.000	-0.000
6	0.000	0.000	-0.001	-0.000	-0.001	0.003	0.001	-0.001	-0.001	0.000
7	0.000	0.000	-0.001	-0.000	-0.001	0.002	0.000	-0.000	-0.000	-0.000
8	-0.000	-0.000	-0.001	-0.001	-0.000	0.002	0.001	-0.001	-0.000	0.000
9	0.000	0.000	-0.001	0.001	-0.002	0.002	0.001	-0.001	0.001	-0.000
10	-0.000	-0.000	-0.001	-0.000	-0.001	0.002	0.002	-0.001	-0.000	0.000

TABLE 4-C22-1 Summed Two Step Transition Counts for Records 6 to 10 and the Differences Between the Values of 10 and the Actual Values.

REC	2 STEP COUNTS 1000 WAVES										LEN	HMAX	TMAX
2	1	2	3	4	5	6	7	8	9	10	87.84	7.67	5.93

	1	2	3	4	5	6	7	8	9	10	
1	6	12	6	12	7	12	11	12	13	9	100
2	12	9	9	10	10	13	14	8	8	7	100
3	11	8	14	15	8	11	9	6	7	11	100
4	14	11	6	5	16	10	8	10	11	9	100
5	11	12	9	7	9	12	9	6	13	12	100
6	15	5	18	10	12	10	13	3	8	6	100
7	8	13	9	12	15	10	11	7	6	9	100
8	5	13	11	13	8	5	5	14	9	17	100
9	9	10	5	10	6	10	10	15	10	15	100
10	9	7	13	6	8	9	11	17	15	5	100
	100	100	100	100	99	102	101	98	100	100	1000

COUNTS - 10

	1	2	3	4	5	6	7	8	9	10
1	-4	2	-4	2	-3	2	1	2	3	-1
2	2	-1	-1	0	0	3	4	-2	-2	-3
3	1	-2	4	5	-2	1	-1	-4	-3	1
4	4	1	-4	-5	6	0	-2	0	1	-1
5	1	2	-1	-3	-1	2	-1	-4	3	2
6	5	-5	8	0	2	0	3	-7	-2	-4
7	-2	3	-1	2	5	0	1	-3	-4	-1
8	-5	3	1	3	-2	-5	-5	4	-1	7
9	-1	0	-5	0	-4	0	0	5	0	5
10	-1	-3	3	-4	-2	-1	1	7	5	-5

TABLE 4-C22-2 Sample Two Step Transition Probabilities for Records 6 to 10 and the Departure from 0.10.

2 STEP PROBABILITY 1000 WAVES

	1	2	3	4	5	6	7	8	9	10
1 *	0.060	0.120	0.060	0.120	0.070	0.120	0.110	0.120	0.130	0.090
2 *	0.120	0.090	0.090	0.100	0.100	0.130	0.140	0.080	0.080	0.070
3 *	0.110	0.080	0.140	0.150	0.080	0.110	0.090	0.060	0.070	0.110
4 *	0.140	0.110	0.060	0.050	0.160	0.100	0.080	0.100	0.110	0.090
5 *	0.110	0.120	0.090	0.070	0.090	0.120	0.090	0.060	0.130	0.120
6 *	0.150	0.050	0.180	0.100	0.120	0.100	0.130	0.030	0.080	0.060
7 *	0.080	0.130	0.090	0.120	0.150	0.100	0.110	0.070	0.060	0.090
8 *	0.050	0.130	0.110	0.130	0.080	0.050	0.050	0.140	0.090	0.170
9 *	0.090	0.100	0.050	0.100	0.060	0.130	0.100	0.150	0.100	0.150
10 *	0.090	0.070	0.130	0.060	0.080	0.090	0.110	0.170	0.150	0.050

PROB MATRIX - 0.100

	1	2	3	4	5	6	7	8	9	10
1 *	-0.040	0.020	-0.040	0.020	-0.030	0.020	0.010	0.020	0.030	-0.010
2 *	0.020	-0.010	-0.010	0.000	0.000	0.030	0.040	-0.020	-0.020	-0.030
3 *	0.010	-0.020	0.040	0.050	-0.020	0.010	-0.010	-0.040	-0.030	0.010
4 *	0.040	0.010	-0.040	-0.050	0.060	0.000	-0.020	0.000	0.010	-0.010
5 *	0.010	0.020	-0.010	-0.030	-0.010	0.020	-0.010	-0.040	0.030	0.020
6 *	0.050	-0.050	0.080	0.000	0.020	0.000	0.030	-0.070	-0.020	-0.040
7 *	-0.020	0.030	-0.010	0.020	0.050	0.000	0.010	-0.030	-0.040	-0.010
8 *	-0.050	0.030	0.010	0.030	-0.020	-0.050	-0.050	0.040	-0.010	0.070
9 *	-0.010	0.000	-0.050	0.000	-0.040	0.000	0.000	0.050	0.000	0.050
10 *	-0.010	-0.030	0.030	-0.040	-0.020	-0.010	0.010	0.070	0.050	-0.050

TABLE 4-C22-3 The Square of the Matrix in Table 4-C22-2 and the Differences Between it and 0.10.

PROB MATRIX SQUARED

	1	2	3	4	5	6	7	8	9	10
1	0.102	0.100	0.099	0.098	0.102	0.100	0.101	0.100	0.097	0.101
2	0.101	0.101	0.101	0.101	0.103	0.103	0.103	0.092	0.098	0.098
3	0.104	0.097	0.102	0.099	0.103	0.104	0.101	0.095	0.101	0.095
4	0.097	0.102	0.098	0.100	0.094	0.104	0.102	0.098	0.104	0.102
5	0.101	0.097	0.101	0.099	0.096	0.104	0.105	0.099	0.101	0.096
6	0.102	0.100	0.100	0.105	0.100	0.107	0.102	0.089	0.098	0.098
7	0.104	0.100	0.100	0.096	0.103	0.105	0.103	0.091	0.100	0.096
8	0.100	0.099	0.101	0.096	0.097	0.099	0.098	0.108	0.103	0.100
9	0.097	0.100	0.101	0.098	0.099	0.098	0.100	0.107	0.102	0.100
10	0.094	0.103	0.100	0.108	0.094	0.098	0.096	0.100	0.095	0.112

PROB MATRIX - 0.100

	1	2	3	4	5	6	7	8	9	10
1	0.002	-0.000	-0.001	-0.002	0.002	-0.000	0.001	-0.000	-0.003	0.001
2	0.001	0.001	0.001	0.001	0.003	0.003	0.003	-0.008	-0.002	-0.002
3	0.004	-0.003	0.002	-0.001	0.003	0.004	0.001	-0.005	0.001	-0.005
4	-0.003	0.002	-0.002	-0.000	-0.006	0.004	0.002	-0.002	0.004	0.002
5	0.001	-0.003	0.001	-0.001	-0.004	0.004	0.005	-0.001	0.001	-0.004
6	0.002	0.000	0.000	0.005	0.000	0.007	0.002	-0.011	-0.002	-0.002
7	0.004	0.000	0.000	-0.004	0.003	0.005	0.003	-0.009	0.000	-0.004
8	-0.000	-0.001	0.001	-0.004	-0.003	-0.001	-0.002	0.008	0.003	0.000
9	-0.003	0.000	0.001	-0.002	-0.001	-0.002	-0.000	0.007	0.002	-0.000
10	-0.006	0.003	-0.000	0.008	-0.006	-0.002	-0.004	0.000	-0.005	0.012

TABLE 5-51 One Step Transition Counts and Sample Markov Transition Matrix for Record 51.

REC	1 STEP COUNTS 200 WAVES											LEN	HMAX	TMAX
51		1	2	3	4	5	6	7	8	9	10	35.30	58.94	11.40
1	*	1	2	2	2	2	1	3	4	2	1	* 20		
2	*	7	3	2	4	1	0	1	0	1	1	* 20		
3	*	1	4	4	2	2	2	4	1	0	0	* 20		
4	*	0	4	4	3	2	0	2	3	1	1	* 20		
5	*	0	1	2	3	1	1	4	4	2	2	* 20		
6	*	2	1	1	2	5	3	1	1	3	1	* 20		
7	*	4	3	1	2	3	3	0	2	1	1	* 20		
8	*	1	1	2	2	3	3	0	3	2	3	* 20		
9	*	2	1	2	0	1	4	1	1	4	4	* 20		
10	*	2	0	0	0	1	3	3	1	4	6	* 20		
		20	20	20	20	21	20	19	20	20	20	* 200		

1 STEP PROBABILITY

		1	2	3	4	5	6	7	8	9	10
1	*	0.05	0.10	0.10	0.10	0.10	0.05	0.15	0.20	0.10	0.05
2	*	0.35	0.15	0.10	0.20	0.05	0.00	0.05	0.00	0.05	0.05
3	*	0.05	0.20	0.20	0.10	0.10	0.10	0.20	0.05	0.00	0.00
4	*	0.00	0.20	0.20	0.15	0.10	0.00	0.10	0.15	0.05	0.05
5	*	0.00	0.05	0.10	0.15	0.05	0.05	0.20	0.20	0.10	0.10
6	*	0.10	0.05	0.05	0.10	0.25	0.15	0.05	0.05	0.15	0.05
7	*	0.20	0.15	0.05	0.10	0.15	0.15	0.00	0.10	0.05	0.05
8	*	0.05	0.05	0.10	0.10	0.15	0.15	0.00	0.15	0.10	0.15
9	*	0.10	0.05	0.10	0.00	0.05	0.20	0.05	0.05	0.20	0.20
10	*	0.10	0.00	0.00	0.00	0.05	0.15	0.15	0.05	0.20	0.30

TABLE 5-52 One Step Transition Counts and Sample Markov Transition Matrix for Record 52.

REC	1 STEP COUNTS 200 WAVES											LEN	HMAX	TMAX
52	1	2	3	4	5	6	7	8	9	10		31.33	45.77	12.52
1	2	3	4	0	2	3	1	2	3	0	*	20		
2	2	2	5	0	2	2	4	2	0	1	*	20		
3	1	2	1	6	5	1	1	0	1	2	*	20		
4	1	4	1	3	4	0	3	3	0	1	*	20		
5	5	3	2	3	0	2	1	2	1	1	*	20		
6	1	0	3	1	0	0	5	3	5	2	*	20		
7	2	1	0	2	1	5	2	2	3	2	*	20		
8	2	2	1	3	2	3	0	2	2	3	*	20		
9	3	2	1	0	2	2	2	3	2	3	*	20		
10	1	1	2	2	1	2	2	1	3	5	*	20		
	20	20	20	20	19	20	21	20	20	20	*	200		

1 STEP PROBABILITY

	1	2	3	4	5	6	7	8	9	10
1	0.10	0.15	0.20	0.00	0.10	0.15	0.05	0.10	0.15	0.00
2	0.10	0.10	0.25	0.00	0.10	0.10	0.20	0.10	0.00	0.05
3	0.05	0.10	0.05	0.30	0.25	0.05	0.05	0.00	0.05	0.10
4	0.05	0.20	0.05	0.15	0.20	0.00	0.15	0.15	0.00	0.05
5	0.25	0.15	0.10	0.15	0.00	0.10	0.05	0.10	0.05	0.05
6	0.05	0.00	0.15	0.05	0.00	0.00	0.25	0.15	0.25	0.10
7	0.10	0.05	0.00	0.10	0.05	0.25	0.10	0.10	0.15	0.10
8	0.10	0.10	0.05	0.15	0.10	0.15	0.00	0.10	0.10	0.15
9	0.15	0.10	0.05	0.00	0.10	0.10	0.10	0.15	0.10	0.15
10	0.05	0.05	0.10	0.10	0.05	0.10	0.10	0.05	0.15	0.25

TABLE 5-53 One Step Transition Counts and Sample Markov Transition Matrix for Record 53.

REC	1 STEP COUNTS 200 WAVES										LEN	HMAX	TMAX
53	1	2	3	4	5	6	7	8	9	10	31.43	50.66	12.41
1 *	6	1	4	1	2	4	2	0	0	0 *	20		
2 *	2	1	1	2	6	2	2	1	2	1 *	20		
3 *	0	3	4	0	1	1	3	4	3	1 *	20		
4 *	3	3	3	4	1	3	0	1	1	1 *	20		
5 *	3	6	0	4	2	1	1	2	0	1 *	20		
6 *	0	2	3	2	3	1	2	1	4	2 *	20		
7 *	0	2	4	3	0	0	2	5	2	2 *	20		
8 *	4	0	0	1	1	2	4	1	4	3 *	20		
9 *	1	1	0	2	2	4	2	1	3	4 *	20		
10 *	1	1	1	1	2	1	2	4	2	5 *	20		
	20	20	20	20	20	19	20	20	21	20 *	200		

1 STEP PROBABILITY

	1	2	3	4	5	6	7	8	9	10
1 *	0.30	0.05	0.20	0.05	0.10	0.20	0.10	0.00	0.00	0.00
2 *	0.10	0.05	0.05	0.10	0.30	0.10	0.10	0.05	0.10	0.05
3 *	0.00	0.15	0.20	0.00	0.05	0.05	0.15	0.20	0.15	0.05
4 *	0.15	0.15	0.15	0.20	0.05	0.15	0.00	0.05	0.05	0.05
5 *	0.15	0.30	0.00	0.20	0.10	0.05	0.05	0.10	0.00	0.05
6 *	0.00	0.10	0.15	0.10	0.15	0.05	0.10	0.05	0.20	0.10
7 *	0.00	0.10	0.20	0.15	0.00	0.00	0.10	0.25	0.10	0.10
8 *	0.20	0.00	0.00	0.05	0.05	0.10	0.20	0.05	0.20	0.15
9 *	0.05	0.05	0.00	0.10	0.10	0.20	0.10	0.05	0.15	0.20
10 *	0.05	0.05	0.05	0.05	0.10	0.05	0.10	0.20	0.10	0.25

TABLE 5-54 One Step Transition Counts and Sample Markov Transition Matrix for Record 54.

REC		1 STEP COUNTS 200 WAVES											LEN	HMAX	TMAX
54		1	2	3	4	5	6	7	8	9	10		33.38	68.47	9.46
1	*	2	4	1	1	3	3	2	2	1	1	*	20		
2	*	2	1	4	5	1	1	4	2	0	0	*	20		
3	*	1	1	2	1	5	0	1	3	3	3	*	20		
4	*	2	4	3	1	1	4	2	2	1	0	*	20		
5	*	5	2	1	2	2	1	4	1	1	1	*	20		
6	*	2	1	2	2	2	4	2	1	3	1	*	20		
7	*	3	2	1	2	3	2	1	2	2	2	*	20		
8	*	1	0	2	5	1	1	1	1	4	4	*	20		
9	*	1	1	3	1	2	2	3	1	2	4	*	20		
10	*	1	4	1	0	0	2	1	5	2	4	*	20		
		20	20	20	20	20	20	21	20	19	20	*	200		

1 STEP PROBABILITY

		1	2	3	4	5	6	7	8	9	10
1	*	0.10	0.20	0.05	0.05	0.15	0.15	0.10	0.10	0.05	0.05
2	*	0.10	0.05	0.20	0.25	0.05	0.05	0.20	0.10	0.00	0.00
3	*	0.05	0.05	0.10	0.05	0.25	0.00	0.05	0.15	0.15	0.15
4	*	0.10	0.20	0.15	0.05	0.05	0.20	0.10	0.10	0.05	0.00
5	*	0.25	0.10	0.05	0.10	0.10	0.05	0.20	0.05	0.05	0.05
6	*	0.10	0.05	0.10	0.10	0.10	0.20	0.10	0.05	0.15	0.05
7	*	0.15	0.10	0.05	0.10	0.15	0.10	0.05	0.10	0.10	0.10
8	*	0.05	0.00	0.10	0.25	0.05	0.05	0.05	0.05	0.20	0.20
9	*	0.05	0.05	0.15	0.05	0.10	0.10	0.15	0.05	0.10	0.20
10	*	0.05	0.20	0.05	0.00	0.00	0.10	0.05	0.25	0.10	0.20

TABLE 5-55 One Step Transition Counts and Sample Markov Transition Matrix for Record 55.

REC	1 STEP COUNTS 200 WAVES											LEN	HMAX	TMAX
55	1	2	3	4	5	6	7	8	9	10		32.60	70.35	11.84
1	*	3	3	2	3	4	2	2	0	1	0	*	20	
2	*	4	1	3	2	2	3	2	1	1	1	*	20	
3	*	1	7	2	3	1	1	1	1	1	2	*	20	
4	*	2	3	5	2	0	4	2	0	1	1	*	20	
5	*	2	2	1	3	2	3	1	2	2	2	*	20	
6	*	2	0	2	3	1	1	3	4	3	1	*	20	
7	*	0	2	1	1	3	2	4	4	2	1	*	20	
8	*	2	1	3	1	3	2	2	2	2	2	*	20	
9	*	3	1	0	1	0	3	0	4	3	5	*	20	
10	*	1	0	1	1	4	0	2	2	4	5	*	20	
		20	20	20	20	20	21	19	20	20	20	*	200	

1 STEP PROBABILITY

		1	2	3	4	5	6	7	8	9	10
1	*	0.15	0.15	0.10	0.15	0.20	0.10	0.10	0.00	0.05	0.00
2	*	0.20	0.05	0.15	0.10	0.10	0.15	0.10	0.05	0.05	0.05
3	*	0.05	0.35	0.10	0.15	0.05	0.05	0.05	0.05	0.05	0.10
4	*	0.10	0.15	0.25	0.10	0.00	0.20	0.10	0.00	0.05	0.05
5	*	0.10	0.10	0.05	0.15	0.10	0.15	0.05	0.10	0.10	0.10
6	*	0.10	0.00	0.10	0.15	0.05	0.05	0.15	0.20	0.15	0.05
7	*	0.00	0.10	0.05	0.05	0.15	0.10	0.20	0.20	0.10	0.05
8	*	0.10	0.05	0.15	0.05	0.15	0.10	0.10	0.10	0.10	0.10
9	*	0.15	0.05	0.00	0.05	0.00	0.15	0.00	0.20	0.15	0.25
10	*	0.05	0.00	0.05	0.05	0.20	0.00	0.10	0.10	0.20	0.25

TABLE 5-C-11-1 Summed Transition Counts for Records 51 to 55 and the Difference Between the Value of Ten and the Actual Value.

REC 1 STEP COUNTS 1000 WAVES LEN HMAX TMAX

11 164.04 70.35 11.84

11		1	2	3	4	5	6	7	8	9	10		
1	*	14	13	13	7	13	13	10	8	7	2	*	100
2	*	17	8	15	13	12	8	13	6	4	4	*	100
3	*	4	17	13	12	10	5	10	9	8	8	*	100
4	*	8	18	16	13	8	11	9	9	4	4	*	100
5	*	15	14	6	15	7	8	11	11	6	7	*	100
6	*	7	4	11	10	11	9	13	10	18	7	*	100
7	*	9	10	7	10	10	12	9	15	10	8	*	100
8	*	10	4	8	12	10	11	7	9	14	15	*	100
9	*	10	6	6	4	7	15	8	10	14	20	*	100
10	*	6	6	5	4	8	8	10	13	15	25	*	100
		100	100	100	100	100	100	100	100	100	100	*	1000

COUNTS - 10

		1	2	3	4	5	6	7	8	9	10	
1	*	4	3	3	-3	3	3	0	-2	-3	-8	*
2	*	7	-2	5	3	2	-2	3	-4	-6	-6	*
3	*	-6	7	3	2	0	-5	0	-1	-2	-2	*
4	*	-2	8	6	3	-2	1	-1	-1	-6	-6	*
5	*	5	4	-4	5	-3	-2	1	1	-4	-3	*
6	*	-3	-6	1	0	1	-1	3	0	8	-3	*
7	*	-1	0	-3	0	0	2	-1	5	0	-2	*
8	*	0	-6	-2	2	0	1	-3	-1	4	5	*
9	*	0	-4	-4	-6	-3	5	-2	0	4	10	*
10	*	-4	-4	-5	-6	-2	-2	0	3	5	15	*

TABLE 5-C-11-2 Transition Probability Matrix for 1000 Waves and the Difference Between it and 0.10 for each Element.

1 STEP PROBABILITY 1000 WAVES

	1	2	3	4	5	6	7	8	9	10
1	0.140	0.130	0.130	0.070	0.130	0.130	0.100	0.080	0.070	0.020
2	0.170	0.080	0.150	0.130	0.120	0.080	0.130	0.060	0.040	0.040
3	0.040	0.170	0.130	0.120	0.140	0.050	0.100	0.090	0.080	0.080
4	0.080	0.180	0.160	0.130	0.080	0.110	0.090	0.090	0.040	0.040
5	0.150	0.140	0.060	0.150	0.070	0.080	0.110	0.110	0.060	0.070
6	0.070	0.040	0.110	0.100	0.110	0.090	0.130	0.100	0.180	0.070
7	0.090	0.100	0.070	0.100	0.100	0.120	0.090	0.150	0.100	0.080
8	0.100	0.040	0.080	0.120	0.100	0.110	0.070	0.090	0.140	0.150
9	0.100	0.060	0.060	0.040	0.070	0.150	0.080	0.100	0.140	0.200
10	0.060	0.060	0.050	0.040	0.080	0.080	0.100	0.130	0.150	0.250

PROB MATRIX - 0.100

	1	2	3	4	5	6	7	8	9	10
1	0.040	0.030	0.030	-0.030	0.030	0.030	0.000	-0.020	-0.030	-0.080
2	0.070	-0.020	0.050	0.030	0.020	-0.020	0.030	-0.040	-0.060	-0.060
3	-0.060	0.070	0.030	0.020	0.040	-0.050	0.000	-0.010	-0.020	-0.020
4	-0.020	0.080	0.060	0.030	-0.020	0.010	-0.010	-0.010	-0.060	-0.060
5	0.050	0.040	-0.040	0.050	-0.030	-0.020	0.010	0.010	-0.040	-0.030
6	-0.030	-0.060	0.010	0.000	0.010	-0.010	0.030	0.000	0.080	-0.030
7	-0.010	0.000	-0.030	0.000	0.000	0.020	-0.010	0.050	0.000	-0.020
8	0.000	-0.060	-0.020	0.020	0.000	0.010	-0.030	-0.010	0.040	0.050
9	0.000	-0.040	-0.040	-0.060	-0.030	0.050	-0.020	0.000	0.040	0.100
10	-0.040	-0.040	-0.050	-0.060	-0.020	-0.020	0.000	0.030	0.050	0.150

TABLE 5-C-11-3 The Square of the Matrix in 5-C-11-2 and the Difference Between it and 0.10.

PROB MATRIX SQUARED

	1	2	3	4	5	6	7	8	9	10
1	0.106	0.105	0.107	0.107	0.106	0.098	0.104	0.096	0.093	0.078
2	0.101	0.118	0.109	0.107	0.105	0.099	0.101	0.099	0.087	0.075
3	0.105	0.107	0.104	0.109	0.100	0.095	0.102	0.098	0.088	0.094
4	0.102	0.108	0.114	0.111	0.106	0.093	0.104	0.094	0.089	0.079
5	0.106	0.104	0.109	0.103	0.102	0.103	0.100	0.097	0.091	0.085
6	0.096	0.100	0.093	0.097	0.096	0.105	0.096	0.104	0.105	0.109
7	0.102	0.094	0.099	0.102	0.099	0.102	0.099	0.099	0.104	0.101
8	0.095	0.098	0.095	0.093	0.095	0.103	0.098	0.103	0.107	0.114
9	0.094	0.083	0.089	0.086	0.097	0.102	0.100	0.104	0.119	0.126
10	0.093	0.082	0.082	0.085	0.093	0.102	0.097	0.108	0.118	0.140

PROB MATRIX - 0.100

	1	2	3	4	5	6	7	8	9	10
1	0.006	0.005	0.007	0.007	0.006	-0.002	0.004	-0.004	-0.007	-0.022
2	0.002	0.018	0.009	0.007	0.005	-0.001	0.001	-0.001	-0.013	-0.025
3	0.005	0.007	0.004	0.009	-0.000	-0.005	0.002	-0.002	-0.012	-0.006
4	0.002	0.008	0.014	0.011	0.007	-0.007	0.004	-0.006	-0.011	-0.021
5	0.006	0.004	0.009	0.003	0.002	0.003	0.000	-0.003	-0.009	-0.015
6	-0.004	0.000	-0.007	-0.003	-0.004	0.005	-0.004	0.004	0.005	0.009
7	0.002	-0.006	-0.001	0.002	-0.001	0.002	-0.001	-0.001	0.004	0.001
8	-0.005	-0.002	-0.005	-0.007	-0.004	0.003	-0.002	0.003	0.007	0.014
9	-0.006	-0.017	-0.011	-0.014	-0.003	0.002	0.000	0.004	0.019	0.026
10	-0.007	-0.018	-0.018	-0.015	-0.007	0.002	-0.003	0.008	0.018	0.040

TABLE 5-C-11-4 The Cube of the Matrix in 5- C-11-2 and the Corresponding Differences.

PROB MATRIX CUBED

	1	2	3	4	5	6	7	8	9	10
1	0.101	0.103	0.102	0.102	0.101	0.100	0.100	0.099	0.097	0.094
2	0.102	0.103	0.103	0.103	0.101	0.099	0.101	0.098	0.096	0.093
3	0.101	0.102	0.102	0.101	0.101	0.099	0.100	0.099	0.097	0.096
4	0.101	0.104	0.103	0.103	0.101	0.099	0.100	0.099	0.096	0.094
5	0.101	0.102	0.102	0.102	0.101	0.099	0.100	0.099	0.098	0.095
6	0.100	0.098	0.099	0.099	0.100	0.100	0.100	0.100	0.102	0.103
7	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100
8	0.099	0.098	0.098	0.098	0.099	0.100	0.100	0.101	0.102	0.104
9	0.098	0.096	0.096	0.096	0.098	0.101	0.099	0.102	0.105	0.108
10	0.098	0.095	0.095	0.095	0.098	0.101	0.099	0.102	0.106	0.111

PROB MATRIX - 0.100

	1	2	3	4	5	6	7	8	9	10
1	0.001	0.003	0.002	0.002	0.001	-0.000	0.000	-0.001	-0.003	-0.006
2	0.002	0.003	0.003	0.003	0.001	-0.001	0.001	-0.002	-0.004	-0.007
3	0.001	0.002	0.002	0.001	0.001	-0.001	0.000	-0.001	-0.003	-0.004
4	0.001	0.004	0.003	0.003	0.001	-0.001	0.000	-0.001	-0.004	-0.006
5	0.001	0.002	0.002	0.002	0.001	-0.001	0.000	-0.001	-0.002	-0.005
6	-0.000	-0.002	-0.001	-0.001	-0.000	0.000	-0.000	0.000	0.002	0.003
7	-0.000	0.000	-0.000	-0.000	-0.000	0.000	-0.000	0.000	0.000	0.000
8	-0.001	-0.002	-0.002	-0.002	-0.001	0.000	-0.000	0.001	0.002	0.004
9	-0.002	-0.004	-0.004	-0.004	-0.002	0.001	-0.001	0.002	0.005	0.008
10	-0.002	-0.005	-0.005	-0.005	-0.002	0.001	-0.001	0.002	0.006	0.011

TABLE 5-C112-1 Summed Two Step Transition Counts for Records 51 to 55 and the Difference Between the Value of 10 and the Actual Value.

REC 2 STEP COUNTS 1000 WAVES

REC											LEN	HMAX	TMAX
11	1	2	3	4	5	6	7	8	9	10	164.00	70.35	11.34

	1	2	3	4	5	6	7	8	9	10	
1 *	7	7	19	10	10	10	12	9	12	4	* 100
2 *	9	12	8	13	11	6	9	12	10	10	* 100
3 *	14	9	7	19	8	7	8	10	9	9	* 100
4 *	8	20	10	7	8	14	7	8	7	11	* 100
5 *	8	8	7	9	17	13	10	7	12	9	* 100
6 *	9	12	9	4	11	10	8	14	13	10	* 100
7 *	7	11	10	8	6	10	18	12	6	12	* 100
8 *	15	8	10	10	9	12	9	12	9	6	* 100
9 *	11	5	8	10	10	12	6	9	8	21	* 100
10 *	11	9	12	10	10	6	12	8	14	8	* 100
	99	101	100	100	100	100	99	101	100	100	* 1000

COUNTS - 10

	1	2	3	4	5	6	7	8	9	10	
1 *	-3	-3	9	0	0	0	2	-1	2	-6	*
2 *	-1	2	-2	3	1	-4	-1	2	0	0	*
3 *	4	-1	-3	9	-2	-3	-2	0	-1	-1	*
4 *	-2	10	0	-3	-2	4	-3	-2	-3	1	*
5 *	-2	-2	-3	-1	7	3	0	-3	2	-1	*
6 *	-1	2	-1	-6	1	0	-2	4	3	0	*
7 *	-3	1	0	-2	-4	0	8	2	-4	2	*
8 *	5	-2	0	0	-1	2	-1	2	-1	-4	*
9 *	1	-5	-2	0	0	2	-4	-1	-2	11	*
10 *	1	-1	2	0	0	-4	2	-2	4	-2	*

TABLE 5-C112-2 Sample Two Step Transition Probabilities for Records 51 to 55 and the Departure from 0.10.

2 STEP PROBABILITY 1000 WAVES

	1	2	3	4	5	6	7	8	9	10
1	0.070	0.070	0.190	0.100	0.100	0.100	0.120	0.090	0.120	0.040
2	0.090	0.120	0.080	0.130	0.110	0.060	0.090	0.120	0.100	0.100
3	0.140	0.090	0.070	0.190	0.080	0.070	0.080	0.100	0.090	0.090
4	0.080	0.200	0.100	0.070	0.080	0.140	0.070	0.080	0.070	0.110
5	0.080	0.080	0.070	0.090	0.170	0.130	0.100	0.070	0.120	0.090
6	0.090	0.120	0.090	0.040	0.110	0.100	0.080	0.140	0.130	0.100
7	0.070	0.110	0.100	0.080	0.060	0.100	0.180	0.120	0.060	0.120
8	0.150	0.080	0.100	0.100	0.090	0.120	0.090	0.120	0.090	0.060
9	0.110	0.050	0.080	0.100	0.100	0.120	0.060	0.090	0.080	0.210
10	0.110	0.090	0.120	0.100	0.100	0.060	0.120	0.080	0.140	0.080

PROB MATRIX - 0.100

	1	2	3	4	5	6	7	8	9	10
1	-0.030	-0.030	0.090	0.000	0.000	0.000	0.020	-0.010	0.020	-0.060
2	-0.010	0.020	-0.020	0.030	0.010	-0.040	-0.010	0.020	0.000	0.000
3	0.040	-0.010	-0.030	0.090	-0.020	-0.030	-0.020	0.000	-0.010	-0.010
4	-0.020	0.100	0.000	-0.030	-0.020	0.040	-0.030	-0.020	-0.030	0.010
5	-0.020	-0.020	-0.030	-0.010	0.070	0.030	0.000	-0.030	0.020	-0.010
6	-0.010	0.020	-0.010	-0.060	0.010	0.000	-0.020	0.040	0.030	0.000
7	-0.030	0.010	0.000	-0.020	-0.040	0.000	0.080	0.020	-0.040	0.020
8	0.050	-0.020	0.000	0.000	-0.010	0.020	-0.010	0.020	-0.010	-0.040
9	0.010	-0.050	-0.020	0.000	0.000	0.020	-0.040	-0.010	-0.020	0.110
10	0.010	-0.010	0.020	0.000	0.000	-0.040	0.020	-0.020	0.040	-0.020

TABLE 5-C112-3 The Square of the Matrix in Table 5-C112-2 and the Differences Between it and 0.10.

PROB MATRIX SQUARED

	1	2	3	4	5	6	7	8	9	10
1	0.102	0.100	0.094	0.107	0.097	0.101	0.097	0.102	0.095	0.105
2	0.099	0.103	0.099	0.100	0.101	0.102	0.098	0.099	0.098	0.100
3	0.096	0.109	0.106	0.097	0.098	0.105	0.097	0.098	0.098	0.098
4	0.099	0.103	0.097	0.102	0.102	0.093	0.097	0.105	0.104	0.098
5	0.096	0.099	0.097	0.095	0.106	0.104	0.098	0.100	0.102	0.104
6	0.103	0.093	0.098	0.102	0.103	0.099	0.097	0.103	0.102	0.101
7	0.099	0.103	0.100	0.100	0.094	0.096	0.107	0.105	0.097	0.098
8	0.099	0.100	0.104	0.099	0.100	0.102	0.098	0.102	0.100	0.096
9	0.100	0.100	0.105	0.096	0.102	0.098	0.100	0.098	0.108	0.095
10	0.099	0.098	0.100	0.104	0.099	0.101	0.100	0.099	0.097	0.105

PROB MATRIX - 0.100

	1	2	3	4	5	6	7	8	9	10
1	0.002	0.000	-0.006	0.007	-0.003	0.001	-0.003	0.002	-0.005	0.005
2	-0.001	0.003	-0.001	0.000	0.001	0.002	-0.002	-0.001	-0.002	0.000
3	-0.004	0.009	0.006	-0.003	-0.002	0.005	-0.003	-0.002	-0.002	-0.002
4	-0.001	0.003	-0.003	0.002	0.002	-0.007	-0.003	0.005	0.004	-0.002
5	-0.004	-0.001	-0.003	-0.005	0.006	0.004	-0.002	-0.000	0.002	0.004
6	0.003	-0.007	-0.002	0.002	0.003	-0.001	-0.003	0.003	0.002	0.001
7	-0.001	0.003	-0.000	-0.000	-0.006	-0.004	0.007	0.005	-0.003	-0.002
8	-0.001	0.000	0.004	-0.001	-0.000	0.002	-0.002	0.002	0.000	-0.004
9	-0.000	0.000	0.005	-0.004	0.002	-0.002	-0.000	-0.002	0.008	-0.005
10	-0.001	-0.002	-0.000	0.004	-0.001	0.001	-0.000	-0.001	-0.003	0.005

TABLE 5-C113-1 Summed Three Step Transition Counts for Records 51 to 55 and the Difference Between the Value of 10 and the Actual Value.

REC 3 STEP COUNTS 1000 WAVES LEN HMAX TMAX

11		1	2	3	4	5	6	7	8	9	10	164.04	70.35	11.84
1	*	11	12	15	7	8	11	11	11	5	9	* 100		
2	*	8	13	9	4	10	12	13	8	11	12	* 100		
3	*	9	15	12	10	11	6	11	9	8	9	* 100		
4	*	10	12	9	8	12	11	6	7	13	12	* 100		
5	*	12	10	5	5	15	12	14	9	14	4	* 100		
6	*	9	5	8	13	7	9	13	13	9	14	* 100		
7	*	8	6	10	17	10	8	7	11	14	9	* 100		
8	*	9	10	7	12	12	9	10	11	10	10	* 100		
9	*	12	7	12	11	8	10	12	13	6	9	* 100		
10	*	11	10	13	13	7	11	3	9	11	12	* 100		
		99	100	100	100	100	99	100	101	101	100	* 1000		

COUNTS - 10

		1	2	3	4	5	6	7	8	9	10	
1	*	1	2	5	-3	-2	1	1	1	-5	-1	*
2	*	-2	3	-1	-6	0	2	3	-2	1	2	*
3	*	-1	5	2	0	1	-4	1	-1	-2	-1	*
4	*	0	2	-1	-2	2	1	-4	-3	3	2	*
5	*	2	0	-5	-5	5	2	4	-1	4	-6	*
6	*	-1	-5	-2	3	-3	-1	3	3	-1	4	*
7	*	-2	-4	0	7	0	-2	-3	1	4	-1	*
8	*	-1	0	-3	2	2	-1	0	1	0	0	*
9	*	2	-3	2	1	-2	0	2	3	-4	-1	*
10	*	1	0	3	3	-3	1	-7	-1	1	2	*

TABLE 5-C113-2 Sample Three Step Transition Probabilities for Records 51 to 55 and Departure from 0.10.

3 STEP PROBABILITY 1000 WAVES

	1	2	3	4	5	6	7	8	9	10
1	0.110	0.120	0.150	0.070	0.080	0.110	0.110	0.110	0.050	0.090
2	0.080	0.130	0.090	0.040	0.100	0.120	0.130	0.080	0.110	0.120
3	0.090	0.150	0.120	0.100	0.110	0.060	0.110	0.090	0.080	0.090
4	0.100	0.120	0.090	0.080	0.120	0.110	0.060	0.070	0.130	0.120
5	0.120	0.100	0.050	0.050	0.150	0.120	0.140	0.090	0.140	0.040
6	0.090	0.050	0.080	0.130	0.070	0.090	0.130	0.130	0.090	0.140
7	0.080	0.060	0.100	0.170	0.100	0.080	0.070	0.110	0.140	0.090
8	0.090	0.100	0.070	0.120	0.120	0.090	0.100	0.110	0.100	0.100
9	0.120	0.070	0.120	0.110	0.080	0.100	0.120	0.130	0.060	0.090
10	0.110	0.100	0.130	0.130	0.070	0.110	0.030	0.090	0.110	0.120

PROB MATRIX - 0.100

	1	2	3	4	5	6	7	8	9	10
1	0.010	0.020	0.050	-0.030	-0.020	0.010	0.010	0.010	-0.050	-0.010
2	-0.020	0.030	-0.010	-0.060	0.000	0.020	0.030	-0.020	0.010	0.020
3	-0.010	0.050	0.020	0.000	0.010	-0.040	0.010	-0.010	-0.020	-0.010
4	0.000	0.020	-0.010	-0.020	0.020	0.010	-0.040	-0.030	0.030	0.020
5	0.020	0.000	-0.050	-0.050	0.050	0.020	0.040	-0.010	0.040	-0.060
6	-0.010	-0.050	-0.020	0.030	-0.030	-0.010	0.030	0.030	-0.010	0.040
7	-0.020	-0.040	0.000	0.070	0.000	-0.020	-0.030	0.010	0.040	-0.010
8	-0.010	0.000	-0.030	0.020	0.020	-0.010	0.000	0.010	0.000	0.000
9	0.020	-0.030	0.020	0.010	-0.020	0.000	0.020	0.030	-0.040	-0.010
10	0.010	0.000	0.030	0.030	-0.030	0.010	-0.070	-0.010	0.010	0.020

TABLE 5-C113-3 The Square of the Matrix in Table 5C113-2 and the Differences Between it and 0.10.

PROB MATRIX SQUARED

	1	2	3	4	5	6	7	8	9	10
1	0.096	0.103	0.101	0.101	0.100	0.096	0.101	0.100	0.100	0.101
2	0.098	0.096	0.100	0.103	0.097	0.099	0.102	0.103	0.102	0.101
3	0.098	0.104	0.099	0.096	0.103	0.100	0.101	0.098	0.104	0.099
4	0.101	0.100	0.101	0.096	0.098	0.101	0.103	0.101	0.099	0.100
5	0.100	0.093	0.097	0.100	0.101	0.100	0.108	0.105	0.101	0.095
6	0.099	0.098	0.101	0.108	0.099	0.098	0.092	0.101	0.103	0.102
7	0.101	0.101	0.099	0.099	0.102	0.099	0.098	0.100	0.101	0.100
8	0.100	0.099	0.098	0.099	0.102	0.101	0.099	0.100	0.104	0.099
9	0.098	0.101	0.101	0.103	0.101	0.097	0.098	0.101	0.101	0.100
10	0.100	0.104	0.103	0.097	0.098	0.099	0.099	0.100	0.096	0.103

PROB MATRIX - 0.100

	1	2	3	4	5	6	7	8	9	10
1	-0.004	0.003	0.001	0.001	-0.000	-0.004	0.001	0.000	0.000	0.001
2	-0.002	-0.004	0.000	0.003	-0.003	-0.001	0.002	0.003	0.002	0.001
3	-0.002	0.004	-0.001	-0.004	0.003	-0.000	0.001	-0.002	0.004	-0.001
4	0.001	-0.000	0.001	-0.004	-0.002	0.001	0.003	0.001	-0.001	-0.000
5	0.000	-0.007	-0.003	-0.000	0.001	0.000	0.008	0.005	0.001	-0.005
6	-0.001	-0.002	0.001	0.008	-0.001	-0.002	-0.008	0.001	0.003	0.002
7	0.001	0.001	-0.001	-0.001	0.002	-0.001	-0.002	0.000	0.001	-0.000
8	-0.000	-0.001	-0.002	-0.001	0.002	0.001	-0.001	0.000	0.004	-0.001
9	-0.002	0.001	0.001	0.003	0.001	-0.003	-0.002	0.001	0.001	0.000
10	0.000	0.004	0.003	-0.003	-0.002	-0.001	-0.001	-0.000	-0.003	0.003

TABLE 6-1 One Step Counts for the 1000 Wave Sample from Records 1 to 5 and the Difference Array.

REC 1 STEP COUNTS 1000 WAVES

REC		1	2	3	4	5	6	7	8	9	10		LEN	HMAX	TMAX
1													80.30	5.69	4.78
1	*	14	13	11	8	10	11	10	6	10	7	*	100		
2	*	17	15	11	11	11	4	8	8	6	9	*	100		
3	*	11	10	15	13	5	10	12	10	9	5	*	100		
4	*	9	9	16	8	15	8	9	10	.8	8	*	100		
5	*	9	6	11	12	10	12	9	11	10	10	*	100		
6	*	12	11	13	7	6	12	6	14	11	8	*	100		
7	*	10	11	7	5	11	11	7	12	13	13	*	100		
8	*	2	10	3	15	13	10	15	6	17	9	*	100		
9	*	10	8	8	12	8	10	7	9	7	21	*	100		
10	*	7	6	6	9	10	13	17	13	9	10	*	100		
		101	99	101	100	99	101	100	99	100	100	*	1000		

COUNTS - 10

		1	2	3	4	5	6	7	8	9	10	
1	*	4	3	1	-2	0	1	0	-4	0	-3	*
2	*	7	5	1	1	1	-6	-2	-2	-4	-1	*
3	*	1	0	5	3	-5	0	2	0	-1	-5	*
4	*	-1	-1	6	-2	5	-2	-1	0	-2	-2	*
5	*	-1	-4	1	2	0	2	-1	1	0	0	*
6	*	2	1	3	-3	-4	2	-4	4	1	-2	*
7	*	0	1	-3	-5	1	1	-3	2	3	3	*
8	*	-8	0	-7	5	3	0	5	-4	7	-1	*
9	*	0	-2	-2	2	-2	0	-3	-1	-3	11	*
10	*	-3	-4	-4	-1	0	3	7	3	-1	0	*

TABLE 6-3 One Step Counts for the 1000 Wave Sample from Records 11 to 15 and the Difference Array.

REC	1 STEP COUNTS 1000 WAVES											LEN	HMAX	TMAX
3		1	2	3	4	5	6	7	8	9	10	100.39	13.33	6.58
1	*	18	6	8	7	5	13	17	11	11	4	* 100		
2	*	12	11	9	14	5	12	8	6	12	11	* 100		
3	*	7	9	10	12	8	11	10	9	12	12	* 100		
4	*	9	14	12	11	10	6	9	10	6	9	* 100		
5	*	10	11	7	11	17	12	7	8	10	7	* 100		
6	*	8	11	11	11	8	13	5	11	9	13	* 100		
7	*	12	14	10	7	4	5	12	11	12	13	* 100		
8	*	5	6	17	9	11	12	10	12	10	8	* 100		
9	*	10	8	9	13	12	8	10	12	8	10	* 100		
10	*	10	11	6	4	16	7	13	10	10	13	* 100		
		101	101	99	99	100	99	101	100	100	100	* 1000		

COUNTS - 10

		1	2	3	4	5	6	7	8	9	10	
1	*	8	-4	-2	-3	-5	3	7	1	1	-6	*
2	*	2	1	-1	4	-5	2	-2	-4	2	1	*
3	*	-3	-1	0	2	-2	1	0	-1	2	2	*
4	*	-1	4	2	1	0	-4	-1	0	-4	-1	*
5	*	0	1	-3	1	7	2	-3	-2	0	-3	*
6	*	-2	1	1	1	-2	3	-5	1	-1	3	*
7	*	2	4	0	-3	-6	-5	2	1	2	3	*
8	*	-5	-4	7	-1	1	2	0	2	0	-2	*
9	*	0	-2	-1	3	2	-2	0	2	-2	0	*
10	*	0	1	-4	-6	6	-3	3	0	0	3	*

TABLE 6-4 One Step Counts for the 1000 Wave Sample from Records 16 to 20 and the Difference Array.

REC 1 STEP COUNTS 1000 WAVES LEN HMAX TMAX

4 1 2 3 4 5 6 7 8 9 10 106.49 14.70 8.1[illegible]

REC	1	2	3	4	5	6	7	8	9	10	Total
1	11	10	8	11	16	10	9	7	11	7	100
2	12	11	16	12	9	11	8	11	8	2	100
3	7	11	17	8	6	12	10	9	10	10	100
4	11	5	8	8	12	12	13	12	8	11	100
5	5	16	8	13	10	6	9	12	13	8	100
6	8	7	8	11	4	7	15	14	11	15	100
7	14	12	8	6	14	9	10	10	10	7	100
8	12	7	7	11	14	15	10	9	7	8	100
9	12	12	10	9	6	10	11	6	11	13	100
10	7	9	8	12	10	8	5	10	11	20	100
	99	100	98	101	101	100	100	100	100	101	1000

COUNTS - 10

	1	2	3	4	5	6	7	8	9	10
1	1	0	-2	1	6	0	-1	-3	1	-3
2	2	1	6	2	-1	1	-2	1	-2	-8
3	-3	1	7	-2	-4	2	0	-1	0	0
4	1	-5	-2	-2	2	2	3	2	-2	1
5	-5	6	-2	3	0	-4	-1	2	3	-2
6	-2	-3	-2	1	-6	-3	5	4	1	5
7	4	2	-2	-4	4	-1	0	0	0	-3
8	2	-3	-3	1	4	5	0	-1	-3	-2
9	2	2	0	-1	-4	0	1	-4	1	3
10	-3	-1	-2	2	0	-2	-5	0	1	10

TABLE 6-5 One Step Counts for the 1000 Wave Sample from Records 21-25 and the Difference Array.

REC 1 STEP COUNTS 1000 WAVES LEN HMAX TMAX

5	1	2	3	4	5	6	7	8	9	10	110.92	16.96	8.18
1	13	15	9	13	10	10	10	6	6	8	100		
2	14	10	10	4	15	11	11	13	7	5	100		
3	5	10	11	13	9	14	12	9	11	6	100		
4	13	12	13	13	8	7	10	7	7	10	100		
5	10	9	9	9	10	12	8	10	13	10	100		
6	9	10	10	9	13	9	10	6	12	12	100		
7	7	13	11	14	10	7	8	7	17	6	100		
8	10	8	10	10	10	14	7	14	3	14	100		
9	9	7	8	8	8	9	11	12	15	13	100		
10	10	5	10	7	7	7	13	16	9	16	100		
	100	99	101	100	100	100	100	100	100	100	1000		

COUNTS - 10

	1	2	3	4	5	6	7	8	9	10
1	3	5	-1	3	0	0	0	-4	-4	-2
2	4	0	0	-6	5	1	1	3	-3	-5
3	-5	0	1	3	-1	4	2	-1	1	-4
4	3	2	3	3	-2	-3	0	-3	-3	0
5	0	-1	-1	-1	0	2	-2	0	3	0
6	-1	0	0	-1	3	-1	0	-4	2	2
7	-3	3	1	4	0	-3	-2	-3	7	-4
8	0	-2	0	0	0	4	-3	4	-7	4
9	-1	-3	-2	-2	-2	-1	1	2	5	3
10	0	-5	0	-3	-3	-3	3	6	-1	6

TABLE 6-6 One Step Counts for the 1000 Wave Sample from Records 26 to 30 and the Difference Array.

REC	1 STEP COUNTS 1000 WAVES											LEN	HMAX	TMAX
6	1	2	3	4	5	6	7	8	9	10		110.87	17.43	8.87
1	12	13	12	9	10	3	11	11	10	9	*	100		
2	11	7	11	11	13	12	4	17	9	5	*	100		
3	15	7	15	9	5	12	5	8	9	15	*	100		
4	7	13	14	12	1[illegible]	10	12	9	10	2	*	100		
5	15	7	10	13	16	6	15	3	9	6	*	100		
6	15	15	6	6	1[illegible]	6	11	10	8	12	*	100		
7	8	10	9	6	[illegible]	12	7	11	16	14	*	100		
8	6	8	8	10	10	12	12	13	12	9	*	100		
9	6	13	9	11	6	11	13	11	7	13	*	100		
10	5	7	6	12	12	15	11	7	10	15	*	100		
	100	100	100	99	[illegible]	99	101	100	100	100	*	1000		

COUNTS - 10

	1	2	3	4	5	6	7	8	9	10	
1	2	3	2	-1	0	-7	1	1	0	-1	*
2	1	-3	1	1	3	2	-6	7	-1	-5	*
3	5	-3	5	-1	-5	2	-5	-2	-1	5	*
4	-3	3	4	2	[illegible]	0	2	-1	0	-8	*
5	5	-3	0	3	6	-4	5	-7	-1	-4	*
6	5	5	-4	-4	[illegible]	-4	1	0	-2	2	*
7	-2	0	-1	-4	-3	2	-3	1	6	4	*
8	-4	-2	-2	0	0	2	2	3	2	-1	*
9	-4	3	-1	1	-4	1	3	1	-3	3	*
10	-5	-3	-4	2	2	5	1	-3	0	5	*

TABLE 6-7 One Step Counts for the 1000 Wave Sample from Records 31 to 35 and the Difference Array.

REC	1 STEP COUNTS 1000 WAVES											LEN	HMAX	TMAX
7	1	2	3	4	5	6	7	8	9	10		117.13	26.15	7.59
1	8	13	10	13	8	13	11	8	10	6	100			
2	21	5	12	7	12	11	6	11	6	9	100			
3	4	14	11	13	14	7	15	8	10	4	100			
4	10	15	7	10	22	6	6	7	12	5	100			
5	10	12	10	9	9	11	7	10	12	10	100			
6	7	9	10	14	5	6	15	11	11	12	100			
7	11	7	12	11	8	7	11	5	17	11	100			
8	7	11	11	6	11	14	10	12	8	10	100			
9	14	13	9	9	7	10	9	13	3	14	100			
10	8	2	8	8	5	15	10	15	11	18	100			
	100	101	100	99	101	100	100	100	100	99	1000			

COUNTS - 10

	1	2	3	4	5	6	7	8	9	10
1	-2	3	0	3	-2	3	1	-2	0	-4
2	11	-5	2	-3	2	1	-4	1	-4	-1
3	-6	4	1	3	4	-3	5	-2	0	-6
4	0	5	-3	0	12	-4	-4	-3	2	-5
5	0	2	0	-1	-1	1	-3	0	2	0
6	-3	-1	0	4	-5	-4	5	1	1	2
7	1	-3	2	1	-2	-3	1	-5	7	1
8	-3	1	1	-4	1	4	0	2	-2	0
9	4	3	-1	-2	-3	0	-1	3	-7	4
10	-2	-8	-2	-2	-5	5	0	5	1	8

TABLE 6-8 One Step Counts for the 1000 Wave Sample from Records 36 to 40 and the Difference Array.

REC		1 STEP COUNTS 1000 WAVES											LEN	HMAX	TMAX
8		1	2	3	4	5	6	7	8	9	10		122.58	26.02	11.03
1	*	14	10	15	13	10	13	7	5	6	7	*	100		
2	*	13	11	13	14	10	6	5	10	12	6	*	100		
3	*	12	10	16	12	4	14	11	5	9	7	*	100		
4	*	7	11	7	8	15	13	9	13	10	7	*	100		
5	*	13	7	9	10	10	8	16	7	9	11	*	100		
6	*	11	12	8	10	18	8	9	14	5	5	*	100		
7	*	5	18	8	17	0	9	5	8	9	12	*	100		
8	*	11	7	8	3	0	14	13	9	11	15	*	100		
9	*	6	9	11	8	8	9	8	14	13	14	*	100		
10	*	8	4	5	5	6	7	17	15	16	17	*	100		
		100	99	100	100	90	101	100	100	100	101	*	1000		

COUNTS = 10

		1	2	3	4	5	6	7	8	9	10	
1	*	4	0	5	3	0	3	-3	-5	-4	-3	*
2	*	3	1	3	4	0	-4	-5	0	2	-4	*
3	*	2	0	6	2	-6	4	1	-5	-1	-3	*
4	*	-3	1	-3	-2	5	3	-1	3	0	-3	*
5	*	3	-3	-1	0	0	-2	6	-3	-1	1	*
6	*	1	2	-2	0	8	-2	-1	4	-5	-5	*
7	*	-5	8	-2	7	-1	-1	-5	-2	-1	2	*
8	*	1	-3	-2	-7	-1	4	3	-1	1	5	*
9	*	-4	-1	1	-2	-2	-1	-2	4	3	4	*
10	*	-2	-6	-5	-5	-4	-3	7	5	6	7	*

TABLE 6-9 One Step Counts for the 1000 Wave Sample from Records 41 to 45 and the Difference Array.

REC 1 STEP COUNTS 1000 WAVES LEN HMAX TMAX

9		1	2	3	4	5	6	7	8	9	10		139.69	37.09	11.44
1	*	12	8	12	11	11	12	13	10	3	8	*	100		
2	*	17	14	11	10	12	9	6	9	10	2	*	100		
3	*	8	20	8	14	13	10	8	10	5	4	*	100		
4	*	11	11	11	7	7	16	12	10	10	5	*	100		
5	*	12	6	13	15	7	11	10	8	11	7	*	100		
6	*	8	11	5	9	10	13	9	12	13	10	*	100		
7	*	12	13	8	13	12	5	7	10	7	13	*	100		
8	*	8	5	17	10	11	8	9	8	13	11	*	100		
9	*	7	8	9	4	11	7	13	9	14	18	*	100		
10	*	5	4	6	6	6	9	15	14	14	21	*	100		
		100	100	100	99	100	100	102	100	100	99	*	1000		

COUNTS - 10

		1	2	3	4	5	6	7	8	9	10	
1	*	2	-2	2	1	1	2	3	0	-7	-2	*
2	*	7	4	1	0	2	-1	-4	-1	0	-8	*
3	*	-2	10	-2	4	3	0	-2	0	-5	-6	*
4	*	1	1	1	-3	-3	6	2	0	0	-5	*
5	*	2	-4	3	5	-3	1	0	-2	1	-3	*
6	*	-2	1	-5	-1	0	3	-1	2	3	0	*
7	*	2	3	-2	3	2	-5	-3	0	-3	3	*
8	*	-2	-5	7	0	1	-2	-1	-2	3	1	*
9	*	-3	-2	-1	-6	1	-3	3	-1	4	8	*
10	*	-5	-6	-4	-4	-4	-1	5	4	4	11	*

Table 6-10 One Step Counts for the 1000 Wave Sample from Records 46 to 50 and the Difference Array.

REC	1 STEP COUNTS 1000 WAVES											LEN	HMAX	TMAX
10		1	2	3	4	5	6	7	8	9	10	159.85	72.23	12.45
1	*	12	13	20	13	6	9	6	11	7	3	* 100		
2	*	10	10	11	9	11	12	14	9	7	7	* 100		
3	*	14	15	10	12	12	9	6	9	9	4	* 100		
4	*	11	9	13	7	11	15	9	7	10	8	* 100		
5	*	15	11	12	17	7	5	11	9	8	5	* 100		
6	*	10	13	5	17	12	12	10	7	8	6	* 100		
7	*	7	10	10	7	7	18	12	12	6	11	* 100		
8	*	7	9	11	8	13	8	8	12	11	13	* 100		
9	*	9	5	5	6	9	7	12	12	20	15	* 100		
10	*	5	5	3	4	11	5	11	14	14	28	* 100		
		100	100	100	100	99	100	99	102	100	100	* 1000		

COUNTS - 10

		1	2	3	4	5	6	7	8	9	10	
1	*	2	3	10	3	-4	-1	-4	1	-3	-7	*
2	*	0	0	1	-1	1	2	4	-1	-3	-3	*
3	*	4	5	0	2	2	-1	-4	-1	-1	-6	*
4	*	1	-1	3	-3	1	5	-1	-3	0	-2	*
5	*	5	1	2	7	-3	-5	1	-1	-2	-5	*
6	*	0	3	-5	7	2	2	0	-3	-2	-4	*
7	*	-3	0	0	-3	-3	8	2	2	-4	1	*
8	*	-3	-1	1	-2	3	-2	-2	2	1	3	*
9	*	-1	-5	-5	-4	-1	-3	2	2	10	5	*
10	*	-5	-5	-7	-6	1	-5	1	4	4	18	*

These ten by ten transition matrices show that it is virtually impossible to predict the height of the next wave as a member of one of ten sets given the height of a wave that has just passed as a member of one of ten sets. The element of surprize common to most accounts of extreme wave at sea is more understandable on the basis of this kind of analysis. In even a statistically unchanging seaway, the time and place that an extreme wave will occur is essentially unpredictable. It can form itself a few tens to hundreds of meters away from a ship and not have existed at all a few minutes earlier.

To proceed further, the number of states were reduced to three by combining probabilities (or counts) in states 1 to 3, states 4 to 7 and states 8 to 10, which correspond to the lowest 30%, the middle 40% and the highest 30% of the waves. The result is a three by three transition matrix as shown in the first column of matrices in Table 7. The upper left number represents the transition of states 1 to 3, the lowest 30%, to states 1 to 3, again the lowest 30%. The next value in the top row is the transition probability from states 1 to 3 to states 4 to 7; that is from the lowest 30% to the middle 40%. The last value in the top row is the transition probability from the lowest 30% to the highest 30%.

The next row of each matrix shows the transition probabilities of the middle 40% to the lowest 30%, the middle 40% and the highest 30%, and the last row is for the transition probabilities for the highest 30% to the lowest 30%, the middle 40% and the highest 30%. If the waves were completely unpredictable, the values in each row would be very close to 0.3, 0.4 and 0.3 for a large enough sample. The corner elements in these eleven matrices are the most interesting. These sample transition probabilities are graphed in Fig. 19, still with the hypothesis that wave height sequences are a one step Markov process. As in both the Figure and Table 7, for the first sample, the probability that if the wave that is present is among the 30% lowest, the following wave will be among the 30% lowest is 0.390. The probability that a wave among the 30% lowest will be followed by a wave among the 30% highest is 0.233. Similar interpretations are possible for each of the four transition probabilities illustrated in the figure. For samples 2 and 3 there is not much difference between the sample probabilities and 0.3. As the hurricane waves build in height, the probabilities that a high wave will be followed by a high wave and a low wave by a low wave increase to

TABLE 7 Three State One Step Markov Transition Matrices for the Lowest 30%, the Middle 40% and the Highest 30% and the Corresponding Theoretical Two Step and Actual Two Step Matrices.

	1 STEP			1 STEP SQUARED			2 STEP ACTUAL		
#1	.390	.377	.233	.316	.394	.290	.283	.407	.310
	.310	.370	.320	.300	.402	.298	.298	.427	.275
	.200	.463	.337	.288	.403	.308	.323	.354	.323
#2	.347	.360	.293	.302	.400	.298	.290	.440	.270
	.293	.397	.310	.298	.403	.299	.327	.423	.250
	.260	.450	.290	.297	.403	.300	.273	.337	.390
#3	.300	.407	.293	.301	.399	.300	.313	.417	.270
	.323	.380	.297	.300	.400	.300	.313	.372	.315
	.273	.417	.310	.300	.399	.300	.270	.420	.310
#4	.343	.407	.250	.299	.402	.299	.340	.377	.283
	.275	.398	.327	.295	.402	.303	.315	.395	.290
	.280	.403	.317	.296	.402	.302	.233	.434	.333
#5	.323	.440	.237	.304	.403	.293	.317	.423	.260
	.315	.393	.292	.301	.401	.298	.302	.383	.315
	.257	.370	.373	.295	.397	.308	.277	.400	.323
#6	.343	.347	.310	.300	.398	.302	.300	.400	.300
	.323	.402	.275	.303	.397	.300	.330	.408	.262
	.227	.450	.323	.297	.405	.298	.297	.390	.313
#7	.327	.433	.240	.303	.401	.294	.303	.397	.300
	.300	.393	.307	.301	.398	.299	.300	.383	.317
	.277	.377	.346	.297	.398	.302	.303	.424	.273
#8	.380	.397	.223	.311	.403	.286	.343	.370	.287
	.290	.435	.275	.300	.402	.298	.273	.392	.335
	.230	.357	.413	.286	.394	.320	.287	.440	.273
#9	.367	.430	.203	.312	.407	.282	.290	.413	.297
	.303	.407	.290	.301	.401	.298	.318	.395	.287
	.230	.363	.407	.288	.394	.318	.287	.393	.320
#10	.383	.397	.220	.315	.403	.282	.287	.423	.290
	.315	.443	.242	.308	.403	.289	.328	.390	.282
	.197	.340	.463	.273	.386	.340	.280	.383	.337
#11	.380	.433	.187	.318	.409	.273	.307	.410	.283
	.313	.415	.272	.304	.402	.294	.298	.400	.302
	.203	.347	.450	.277	.388	.335	.297	.386	.317

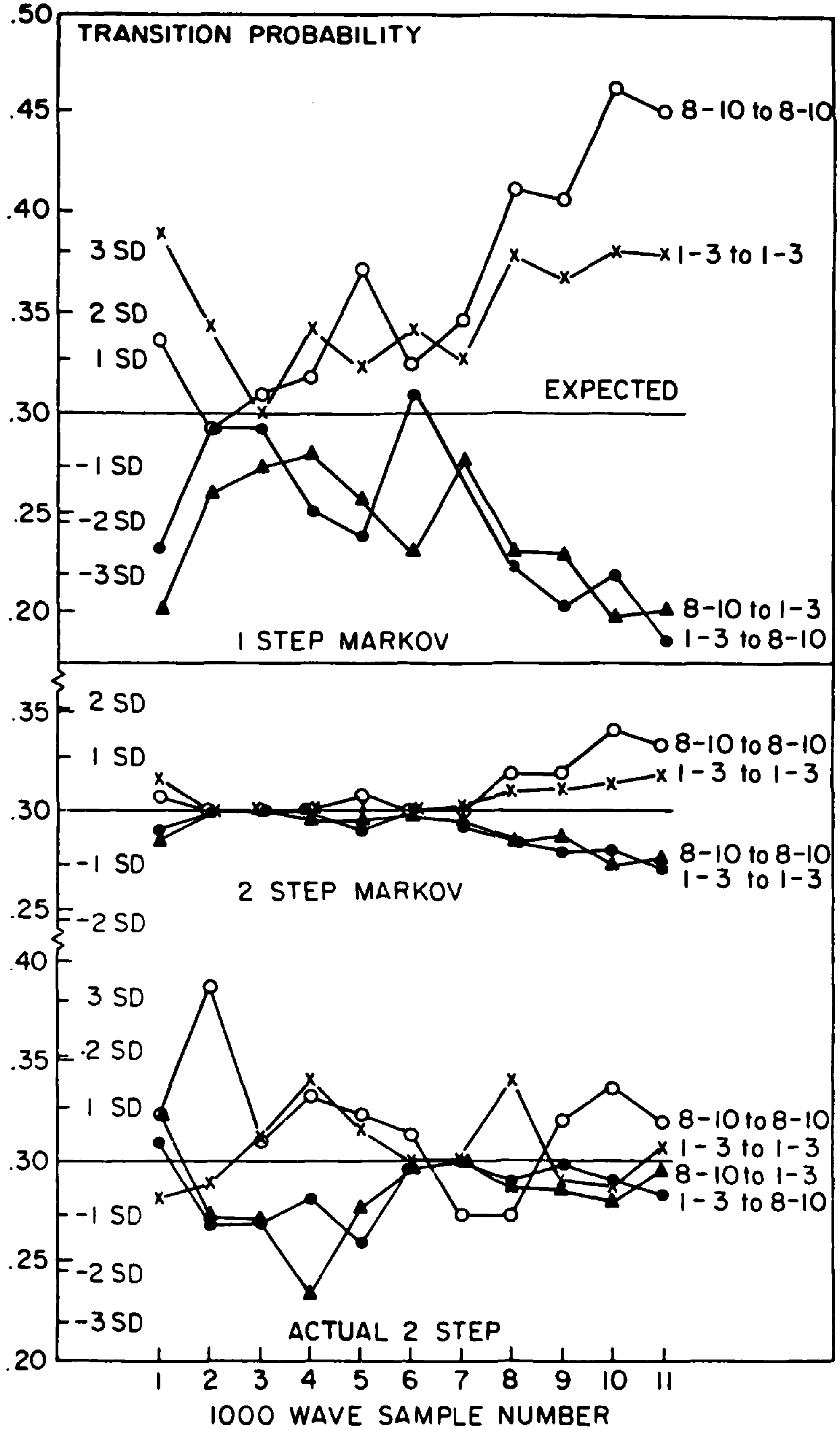

FIGURE 19 Selected Transition Probabilities for an Assumed Three State Markov Process.

values of 0.450 and 0.380. The probabilities that either a low wave will be followed by a high wave or a high wave will be followed by a low wave change from near 0.3 (essentially random) to values of 0.187 and 0.203.

Of 1000 waves, 300 are the lowest 30% and 300 are the highest 30%. If the lower waves had a probability of 0.3, 1000 waves are sampled, and if the waves were random the expected number of transition lower waves would be 300 with a standard deviation of $(0.3 \times 0.7 \times 1000)^{\frac{1}{2}}$ (or $(210)^{\frac{1}{2}}$). Sample probabilities of $(300 \pm p(210)^{\frac{1}{2}})/1000$, since estimated probabilities have sampling variability, are shown on the transition probability scale as plus or minus one to three standard deviations. Toward the end of the full data set, as the hurricane waves build in height, their heights clearly begin to depend upon, at least, the height of the preceding wave.

The square of a one step transition matrix should give the two step transition matrix. These squared matrices are also shown in Table 17. The transition probabilities are also graphed in Fig. 19 for the two step process as labled. Probabilities of 0.318, 0.335, 0.273 and 0.277 are so little different from 0.300 that if one knew the height of a passing wave the second wave to follow could not be predicted better than by chance.

The actual two step transitions are also tabulated and graphed. Most of the four corner values are within one standard deviation and all but two out of forty four are within two standard deviations. Knowledge of the height of a passing wave is of little value in predicting the height of the second wave to follow it.

It is also possible to simplify the prediction but base it more accurately on the heights of the initial wave. As in Figure 20, given that the initial wave is in state 1 (that is one of the lowest 10%) the sample probability that the next wave will be states 1 to 3 or 8 to 10 can be found. If a very low wave has passed, the chance that the next wave will be among the 30% lowest exceeds 0.4 and that it will be among the 30% highest is under 0.2. Similarly if a very high wave has passed, it is better than even the next wave will be among the 30% highest and under 0.2 that it will be among the 30% lowest.

Even 1000 wave samples are somewhat irregular as in Fig. 19. Nine overlapping 3000 wave samples can be formed by averaging the sample

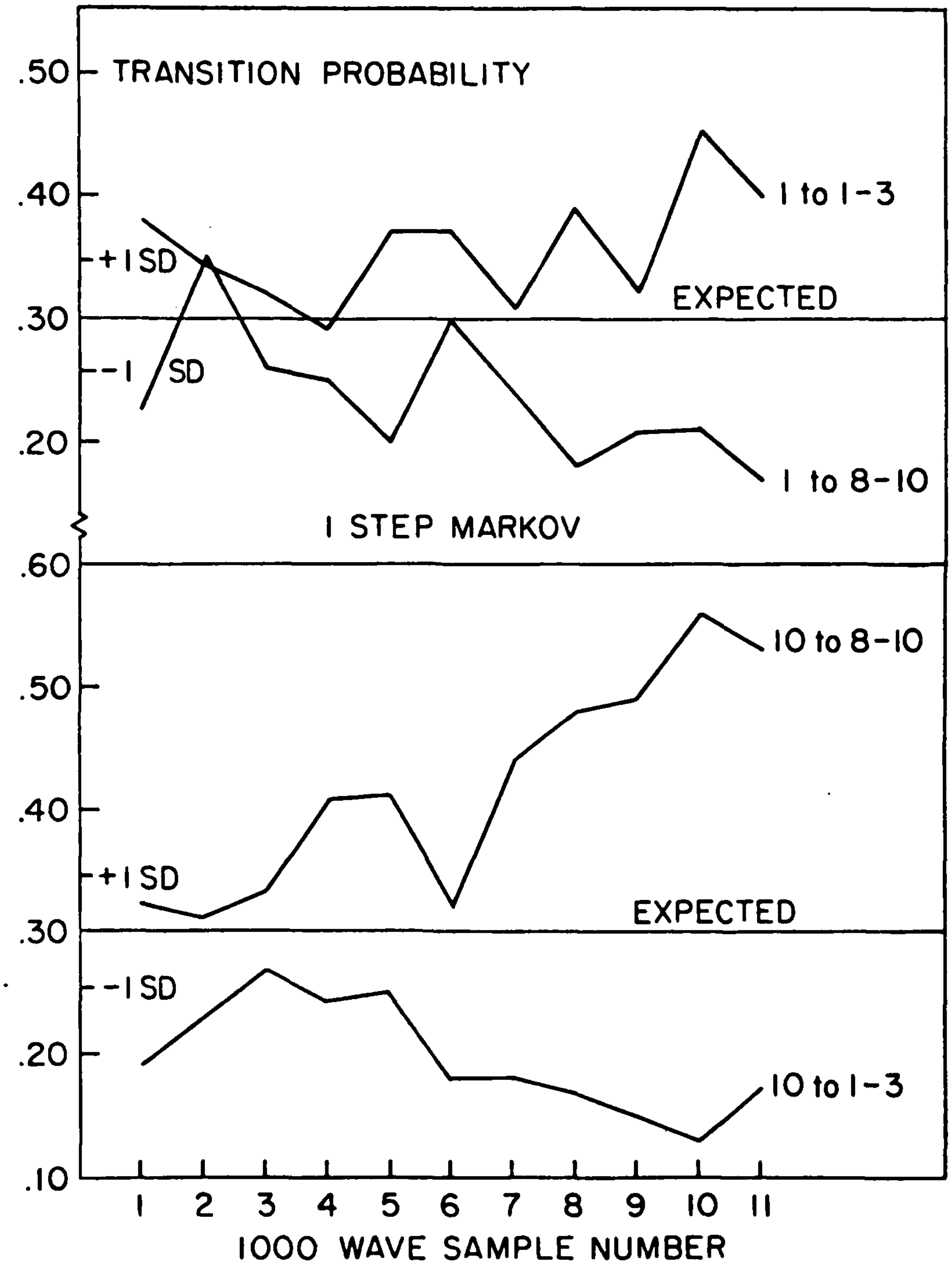

FIGURE 20 Sample Transition Probabilities from States 1 and 10 to States 1 to 3 and States 8 to 10.

probabilities by three in Table 7. For the four corners of the matrices that result the appropriate transition probabilities for a one step Markov and for the actual two step transition are shown in Fig. 21.

Whether or not a Markov process adequately represents the sequence of successive crest to trough heights during Camille is difficult to judge. There are other ways to predict the evolution of a time series that depend on more of the past behavior of the waves than on the height of the wave that just passed. For the present purpose, however, the analysis shows that the second wave to follow a particular wave is nearly independent of the height of that wave and that the height of the first wave to follow a particular wave is not random in the sense required for a random sample.

The two hundred wave samples that are the basis for the analysis of the Camille wave data are not the equivalent of 200 independent samples from a pdf that would be definable in terms of either statistics of the sample or the spectrum of the wave in terms of moments of the spectrum. The equivalent number of independent wave heights is surely less than 200, at least toward the end of the 55 sets of data. It may be somewhat larger than 100 because the dependence of the next wave on the previous wave is weak. Since these sample transition probabilities are a function of sample number, the effective number of independent observations changes from one sample to the next and may decrease as the sample number goes from 1 to 55. The lack of independence will have further repercussions in attempting to interpret the data.

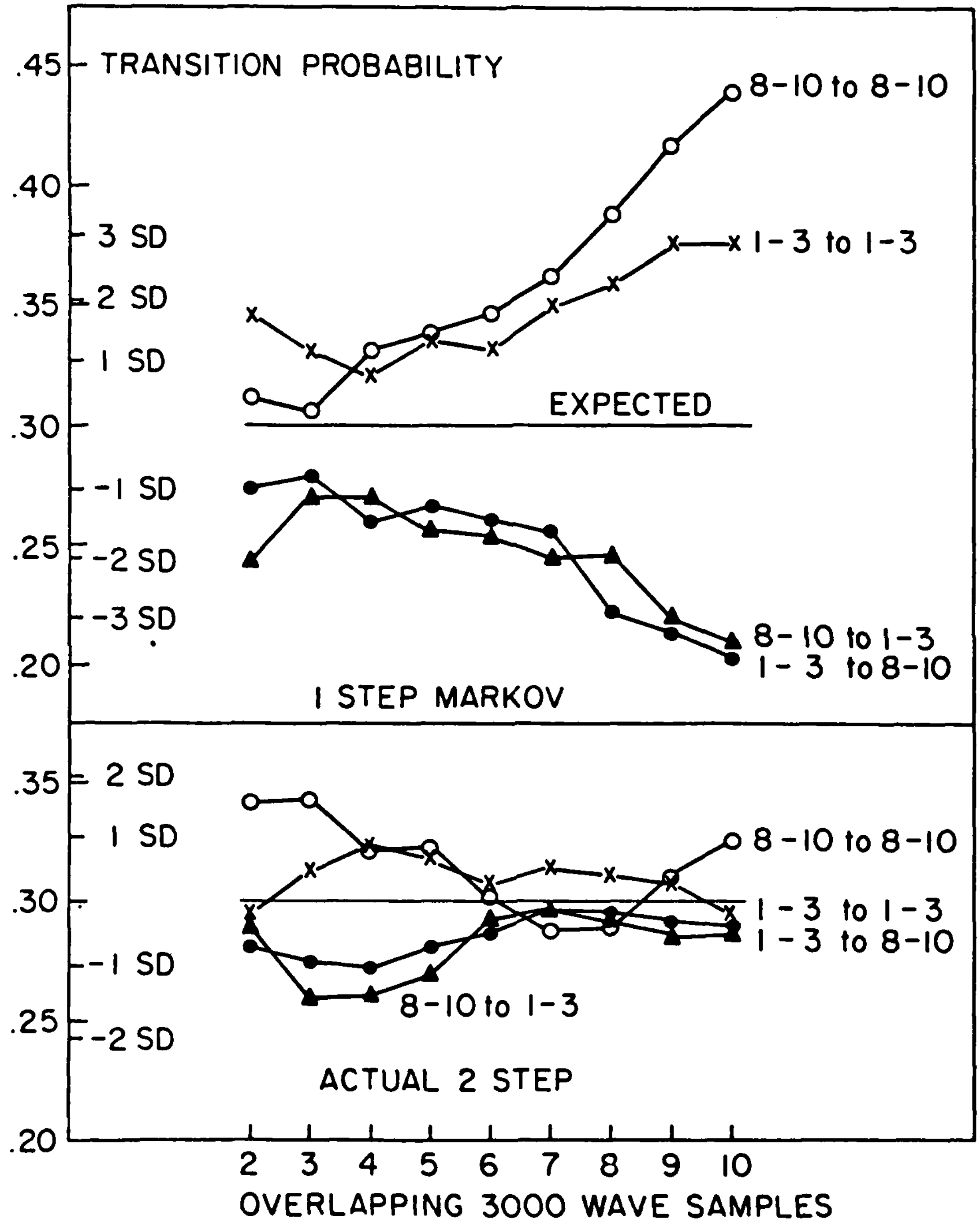

FIG. 21 Transition Probabilities for Overlapping 3000 Wave Samples.

SAMPLE CUMULATIVE DENSITY FUNCTIONS

Introduction. One of the ways to display a sample of crest to trough ocean wave heights is by means of the sample cumulative density function, or a variation thereof. A plot of the probability that h will exceed a given value on a log scale, labled in antilog values, as a function of h on a linear scale is often used, frequently in normalized form as in Fig. 18. If the horizontal scale of the plot is changed to h^2 instead of h, the theoretical Rayleigh cdf becomes a straight line.

One might think that either of these two ways to display the data from a sample provide useful information about the high wave tail of the distribution. At h equal to zero the data and the theory start out at one. The first 90% of the data occupy the first decade of the probability scale; the next 9% occupy the second decade; the next 0.9% the third decade; and next 0.09%, the fourth, and so on.

For a 200 wave sample, the highest wave falls at the antilog point 0.005. It is difficult to judge how well the first 90% of the data fit the theory because of the distortion of the scales. The points for such plots usually fall well below the theoretical transformed Rayleigh shape with actual sample exceedance probabilities often a factor of two or more smaller than the theoretical for the last few points of the plot. It is then tempting to conclude that the Rayleigh distribution does not fit the data well for high waves. This was the conclusion reached in our previously cited unpublished study for DTNSRDC. Such a conclusion is no longer obvious, as the following analysis will show, because of certain inherent properties of sample cdf's and because the sample of wave heights is not a true random sample.

Properties of Sample CDF's. A sample cdf, $F_n(x)$, is defined to be

$$F_n(x) = \frac{1}{n} \text{ (number of } X_i \text{ less than or equal to x)} \qquad (107)$$

where X_1, X_2,.....X_n is a random sample. The function, $F_n(x)$, assumes the values 0, 1/n...2/n...n/n and can be graphed in a stair step pattern. Under these conditions it can be proved that

$$P\left[F_n(x) = \frac{k}{n}\right] = \binom{n}{k}(F(x))^k \left[1 - F(x)\right]^{n-k} \qquad \text{for } k = 0, 1....n \qquad (108)$$

If one knew the population cdf, $F(x)$, then in principle $F_n(x)$ for a particular x can have any of the above fractions as values from zero to one. It can also be shown that

$$\mathcal{E}\ (F_n(x)) = F(x) \tag{109}$$

and $$\mathrm{Var}(F_n(x)) = \frac{1}{n} F(x)\ [1 - F(x)] \tag{110}$$

Everything appears to be straight forward and clear from the sampling theory point of view, but the actual situation is deceptive for typical sample sizes over the full range of definition of $F_n(x)$.

<u>An Analysis of the Consequences of the Properties of Sample CDF's</u> Although the following analysis applies to any cdf, it will be applied to wave heights for illustration. Assume that wave heights are in fact Rayleigh distributed and that E_o is exactly known. Then a normalized variable given by

$$X = h/2(E_o)^{\frac{1}{2}} \tag{111}$$

would have the cdf given by

$$F(x) = 1 - \exp(-x^2/2) \tag{112}$$

Suppose that we have a true random simple of 200 values of X_i. Also to simplify analysis, let $x_{.005}$ be the value of x in (112) that makes $F(x)$ equal to 0.005, that is

$$X_{.005} = -2\ \ell\eta(1 - .005) \tag{113}$$

the numbers,

$$x_{.005}\quad x_{.010}\quad x_{.015}\cdots\quad x_p\cdots\quad x_{0995}$$

can then be computed. For each value of the X_i

if $0 < X_i \leq x_{.005}$ let X_i equal $x_{.005}$ numerically,

if $x_{.005} < X_i \leq x_{.010}$ **let** X_i equal $x_{.010}$ numerically,

if $x_{p-1} < X_i \leq x_p$ let X_i equal x_p numerically,

up to $x_{.995}$ on the right hand side of the inequalities and if

$x_{.995} < X_i < \infty$ let X_i equal its actual value.

The odds are virtually infinitessimal, $(200!)/(200)^{200}$, that there will be exactly one value of X_i in each of the 200 intervals defined above, and consequently the sample cdf will depart in some way from the population cdf.

Figure 22 shows what can happen for low values of X_i, or as transformed, x_p. The circled points are the first 12 values of x_p. The numbers are probabilities times 1000 from (108) for $x = x_p$. If one had the luxury of 1000 random samples of 200, for about 367 samples there would be no x_p in the sample for $x_{.005}$ (numerically $((0.995)^{200}) \times 1000$). There would be 369 jumps to $F_n(x) = 0.005$, there would be 182 jumps to $F_n(x) = 0.010$, 62 to 0.015, 15 to 0.020 and 3 to 0.025. Numbers have been rounded to the nearest integer and adjusted to sum to 1000. Although theoretically possible, there would be less than 1 jump in 2000 for any value greater than 0.025. For 736 times in 1000 the sample cdf will be less than or equal to the population cdf for the first circled point.

For the second circled point, for 134 times there will still be no jump and so on reading the values for increasing F(x). For 405 times, the sample cdf will be below the circled point. For 677 times it will be at or below the circled point. For only 323 times will $F_n(x)$ exceed F(x).

For the next ten points for a total of 12, where F(x) reaches 0.060, the sample cdf has a greater probability of being below the theoretical value than it has of being above it.

A given sample cdf can, in principle, pass through every one of the plotted points (and more) as long as it increases monotonically or stays constant, but most of them start out below F(x) and tend to stay below F(x) at the beginning of the plot of $F_n(x)$.

Fig. 23 shows a simplified version of the previous figure. The points are the values of $F(x_p)$, and the circled points have three numbers plotted near them. The upper number is the chance times 1000 that $F_n(x)$ lies above the plotted point; the middle one is the chance times 1000 of being at the plotted point and the lower one is the chance times 1000 of being below the plotted point.

The high end of the sample cdf is upside down and backward relative to the low end as shown in Fig. 24. The chance that it will be above the population cdf exceeds the chance that it will be below it. The last value, which by definition is plotted at $F_n(x_{1.000}) = 1$, will occur before or at x_p equal to about 3.2 for 367 out of 1000 times. The numbers across the top are the times in 1000 that $F_n(x_p)$ reaches the value 1 at the plotted point. The numbers by some of the points are the times in 1000 that the sample cdf lies above, at, or below the population cdf.

For a collection of random samples known to be from the normalized pdf, more than half of the sample cdf's will be below, or at, the population cdf for small values of the random variable. More than half will also be above, or at, the population cdf for high values of the random variable. Over the entire range of the X_i, those sample cdf's that start out below and end up above the population cdf would be most frequent (more than 25%). Those that start out above and end up below would be least frequent, (less than 25%). Those that either start out above and end up above or start out below and end up below would be about equally divided among the remainder.

The sample cdf in (108) has a symmetric binomial distribution about an expected value of 0.5 only for $k = n/2$ which is the median, corresponding to x equals 1.1774 for the normalized pdf. The mode is at x equals 1. A sample cdf has an equal chance of being above or below the population cdf near $F_n(x) = 0.5$.

From the above analysis, a random sample will tend to be deficient in sample values at both the low end of the range of definition of the random variable and at the high end of its range of definition. With too few sample values for low and high values of the random variable, there must be too many in some intermediate range between the low and the high sample values and the sample cdf will rise more rapidly over this range. The consequence is that the histogram that can be constructed to compare to the population pdf will very often be lower than the values computed from the population pdf at the outer ranges of the distribution and higher in the middle. The result could be a sample pdf that resembled the dots in Fig. 17 whereas in fact the sample came from a population represented by the solid curve.

It appears that many texts that describe the use of the Chi Square test of goodness-of-fit recommend pooling a number of class intervals at the low and high end of the sample distribution, thus giving up some degrees of freedom, before calculating χ^2. If this is not done, the usual consequence is a number of large values in the sum used to compute Chi Square that lead to the rejection of the hypothesis that the sample came from the hypothesised pdf.

Non-Random Samples. The samples of 200 consecutive waves have been shown not to satisfy the definition of a random sample. Each sample might still have come, for practical purposes, from a Rayleigh pdf in the sense that $f_{(x_1, x_2, \ldots x_n)}(x_1, x_2, x_3, \ldots., x_n)$ as on page 79 may take a form, for practical purposes, something like

$$f_{(x_{p-1}, x_p, x_{p+1})}(x_1, x_2, x_3, \ldots., x_n)$$

and this is turn may have a marginal pdf that is Rayleigh. Successive equally spaced points in a time history are strongly correlated, but nevertheless this pdf can be defined and studied.

The difficulties arise in testing whether or not the sample comes from a given population and in trying to study the extreme values of the sample. The concept of an effective number of independent points given a sample of n values is thus useful.

Suppose that by Monte Carlo methods a truly independent sample of 100 random variables is drawn from the normalized Rayleigh pdf. Then, for simplicity, add a small random positive or negative perturbation to each of the X_i to produce a non-random sample of 200. Given this doctored sample of 200 values, a statistician unaware of how it was produced might proceed to analyse it as if it were a random sample of 200 values. The sample cdf would look a bit strange because it would jump twice for values of X_i that were close together but the statistician might not spot this peculiarity.

From the previous results, the chance of not getting a value of X_i less than $x_{.005}$ was $(0.995)^{200}$ or 0.317 for a sample of 200. For our fudged sample, it is $(0.995)^{100}$ or 0.606. The corresponding values for the next three points (200 versus 100) are; 0.134 and 0.366, 0.048 and 0.221, and 0.017 and 0.133.

The chance of not seeing a high wave corresponding to an exceedance probability of 0.005 has almost doubled. The true situation may not be quite that bad, but it is easily seen that the lack of a random sample causes difficulties that are not considered in most of the papers on the subject.

Both the Chi Square test of goodness of fit discussed on page 80 and the distribution of the sample cdf defined by (108), (109) and (110) are for large samples. How large a large sample ought to be is not well defined. For a p of 0.995 and a random sample of 200, the distribution of F(199/200) is skewed and the sample cdf has only a small chance of having the appropriate value. To obtain a realistic sample cdf might require a much larger random sample, say, one ten times greater. Then $\mathcal{E}(F_{1990}(x))$ would still be 0.995 and $VAR(F_n(x))$ would be one tenth of the value for a sample of 200.

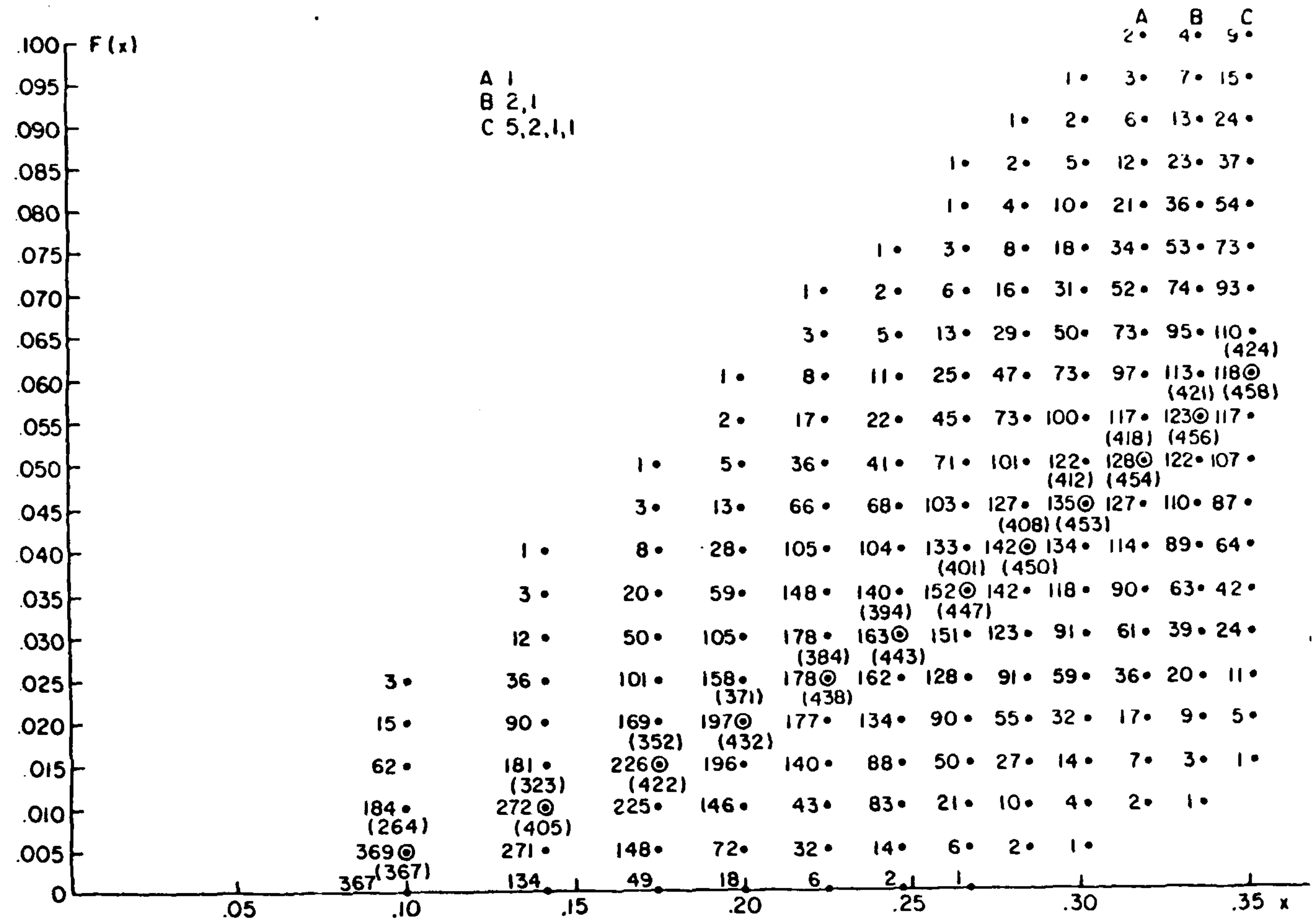

FIGURE 22 Properties of a Sample CDF for Low Values of x_p. The Numbers Beside A, B and C are the Values Associated with the Upward Extentions of Columns A, B, and C.

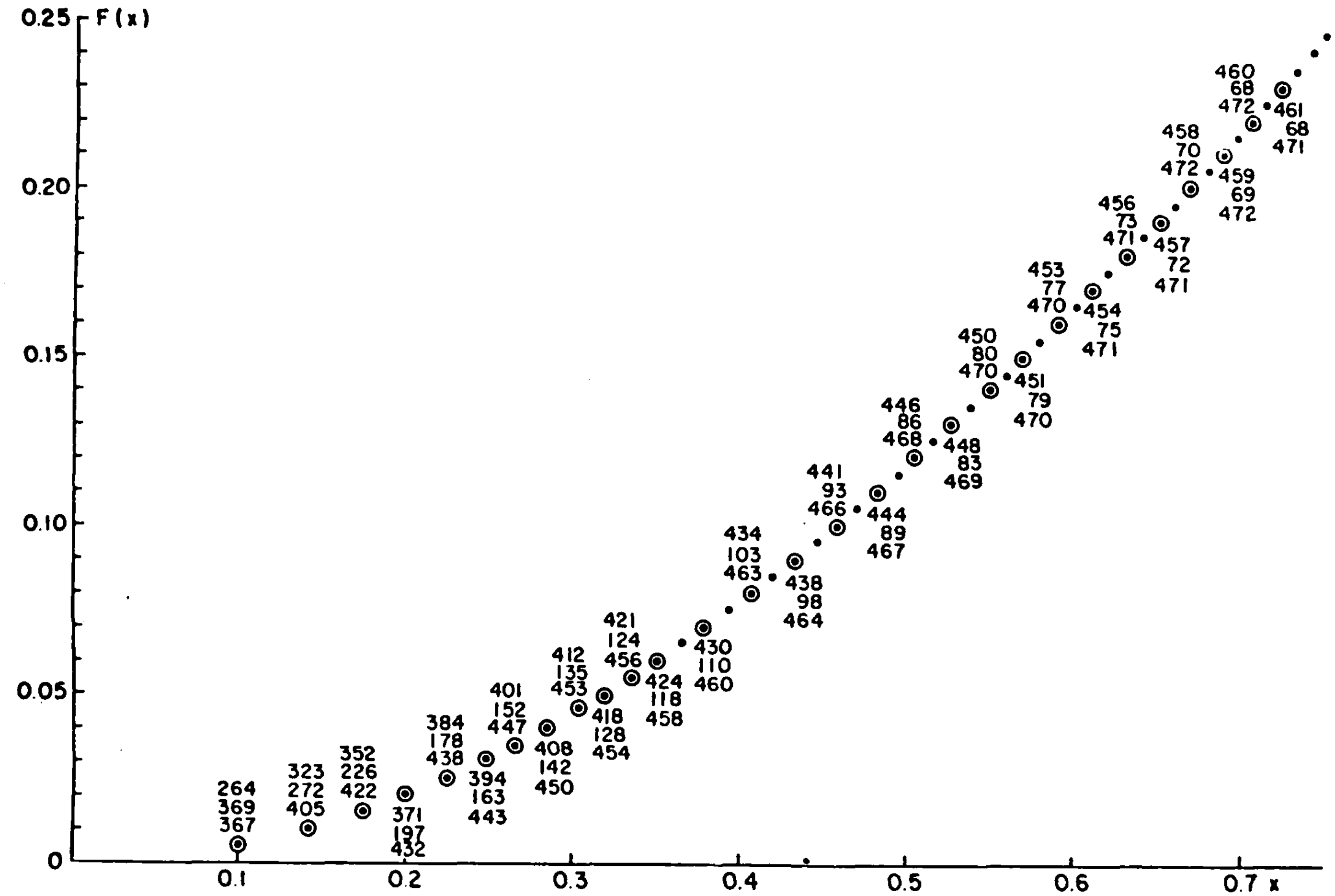

FIGURE 23 Properties of the Sample CDF for Values of x_p Up to a Value Corresponding to $F(x_p)=0.245$.

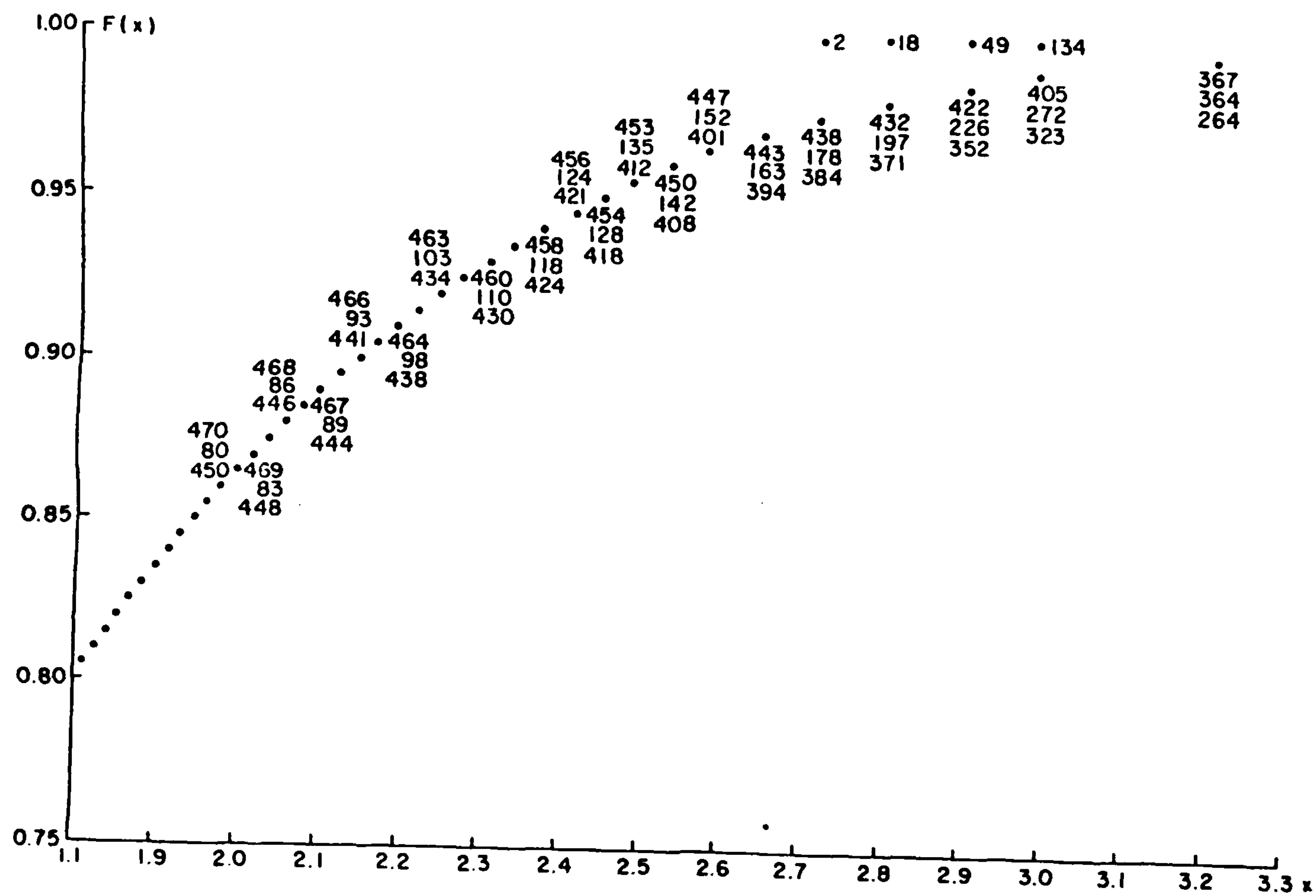

FIGURE 24 Properties of the Sample CDF for Values of x_p at or Past the Value Where $F(x_p) = 0.805$.

COMPUTATIONS BASED ON THE RAYLEIGH DISTRIBUTION

Three models have been suggested as possible reasons for the departure of crest to trough wave heights from the Rayleigh pdf. Two have to do with variation of the envelope between a crest and the following trough. Given the required spectral moments, the modifications to the Rayleigh pdf would be determined by the theoretical analyses given above. The third requires an assumption about the physical behavior of the higher waves in a time history as they break. Each of these three models introduces additional degrees of freedom for fitting a theoretical pdf to a set of observations of crest to trough wave heights. With more freedom as to possible forms for the pdf, it should be expected that a better fit, in some sense, might be obtained.

Is it possible that the Rayleigh pdf does fit the significant wave height part of the actual crest to trough wave heights if the pdf is based on the variance given by the area under the spectrum? The data presented so far shows that the significant wave height computed from the crest to trough heights is nearly always less than the value computed from the area under the spectrum.

This may be a result of the sampling problem that has been described in the preceding section. As shown by the work of Spring (1978), even the additional freedom to pick the value of the one available Rayleigh parameter does not produce agreement between the theory and the data for the goodness-of-fit test that was used. If any of the three models described above are important the actual shape of the pdf would change.

The shape of the pdf and the cdf of the Rayleigh distribution is completely determined by one parameter. Figure 25 shows a set of such cdf's labled with the appropriate significant wave heights. A departure of the observations from the cdf might imply either that the theory is at fault or that the lack of an independent random sample has introduced added complexities.

A variant of the Rayleigh distribution given by

$$F(h) = 1 - e^{-ah^{b}} \tag{114}$$

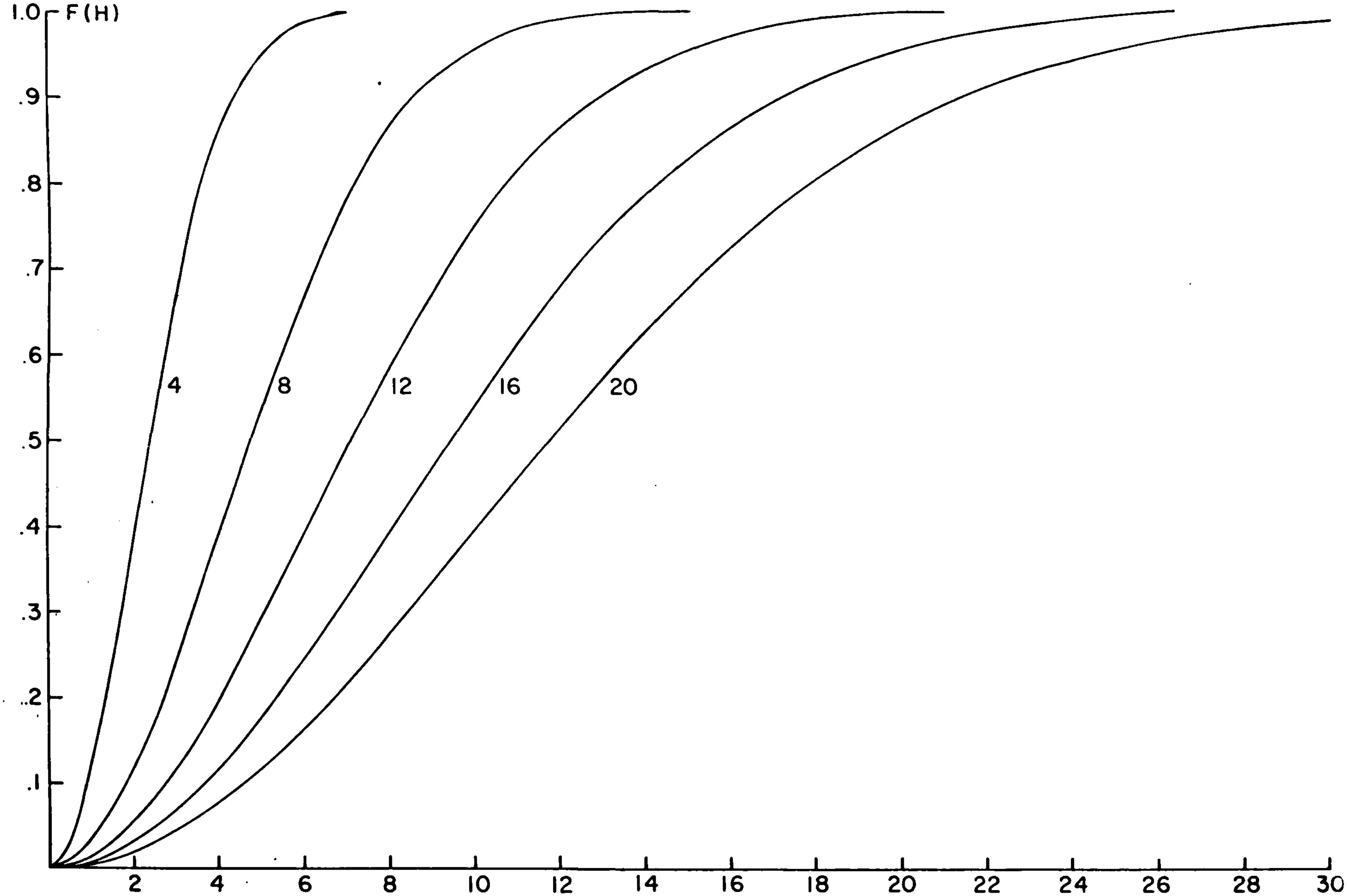

Figure 25 Examples of Cumulative Density Functions for the Rayleigh Distribution Labled with the Significant Wave Height.

This cdf provides two parameters instead of one with which to fit the data. The pdf is given by

$$f(h) = abh^{b-1} e^{-ah^b} \qquad \text{for } 0 < h < \infty \tag{115}$$

and zero otherwise. Fig. 26 illustrates how the Rayleigh cdf can be modified by lowering the curve for $h < h_1$ and raising it for $h > h_1$.* There are no physical or theoretical reasons for such an assumed form for this family of pdf's other than, perhaps, an improved empirical fit. The model actually mimics the kinds of sample cdf's that can result from a typical sample of crest to trough wave heights.

The Rayleigh pdf is a good starting place to try to study the actual distributions of crest to trough wave heights. If consistent systematic departures from the Rayleigh can be demonstrated to occur for wind seas, this could be the first step toward obtaining an improved pdf.

To investigate the possibility of consistent departures from the Rayleigh pdf so as to provide guidance in the interpretation of results to be obtained from the preceeding theories the wave height data have been processed in a number of different ways. Given the value of E_o in (1), the pdf is completely determined.

The average wave height is, of course, given by.

$$\bar{h}_{mean} = \int_o^\infty h\, f(h)dh = 2.5066\, (E_o)^{1/2} \tag{116}$$

The value of α such that

$$\int_\alpha^\infty f(h)\, dh = \frac{1}{3} \tag{117}$$

yields the significant wave height, or the average of the heights of the one third highest waves.

$$\bar{h}\,\frac{1}{3} = 3\int_\alpha^\infty h\, f(h)dh = 4.0046\, (E_o)^{1/2} \tag{118}$$

*Or vice versa.

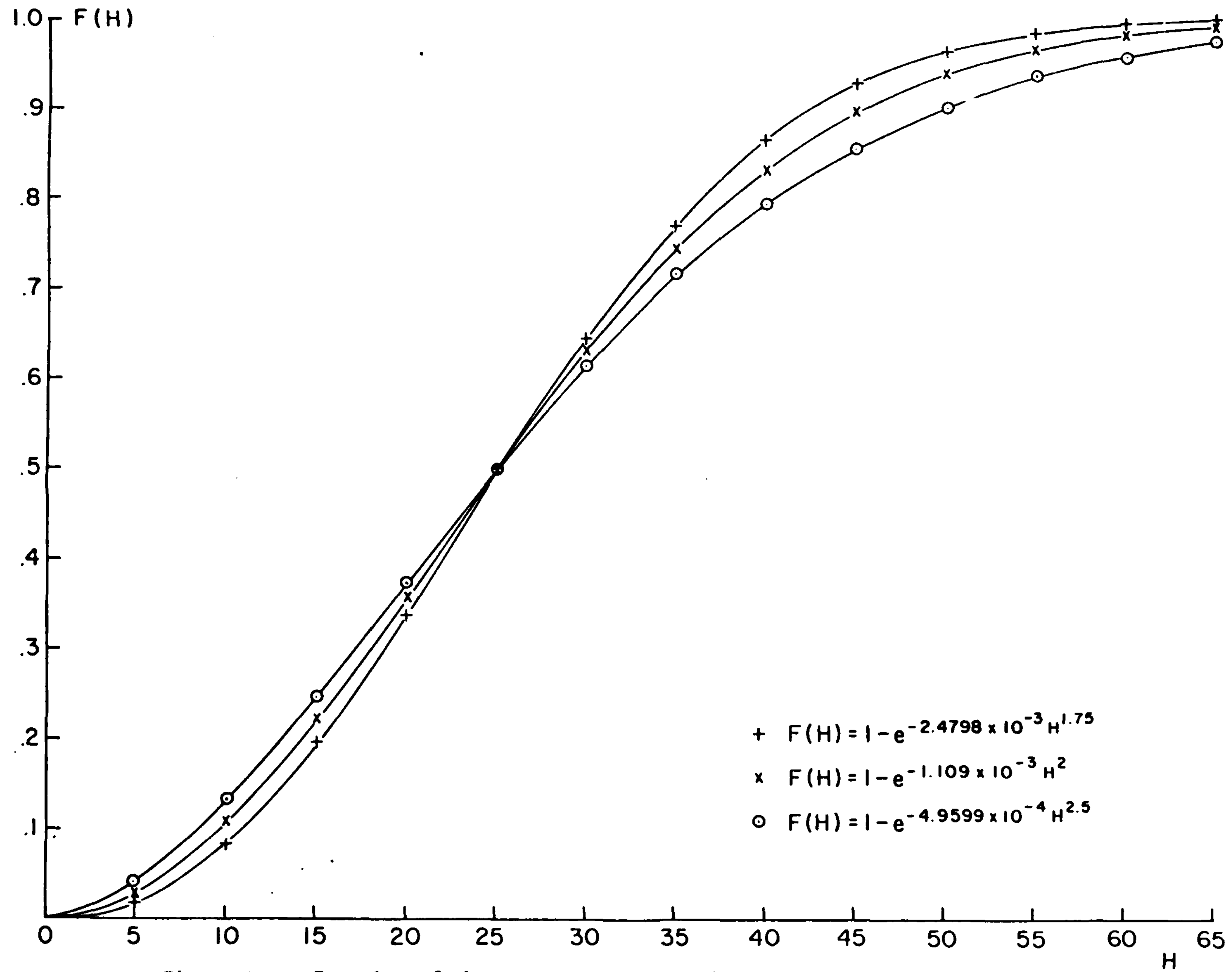

Figure 26 Examples of the Two Parameter Cumulative Distribution for the Values Shown.

Other quantities all based on the Rayleigh pdf such as the average of the heights of the one tenth highest waves, and the expected value of the highest wave in 200 can be found. Also that highest wave, such that for 5% (95%) of a large number of samples of size N, the highest wave will be less (greater) than that highest, can be found (Longuit-Higgins (1952)).

All of these properties of the pdf are represented by some constant times $E_o^{\frac{1}{2}}$. Each therefore provides a way to estimate $E_o^{\frac{1}{2}}$.

If the sample is known to have come from a pdf with a known analytical form but with an unknown parameter (or unknown parameters) then the way to proceed is well known. However, if the problem is to find out how the data differ in some subtle ways from the theoretical pdf, the way to proceed is not as clearly defined. If the additional parameters provide a family of curves that vary smoothy and equal the Rayleigh for zero values of the new parameters, one might always expect, but not necessarily actually have, a better fit.

The data from Hurricane Camille are obviously not from a stationary process. Nor are the successive wave heights independent (or uncorrelated). They key requirement for the analysis of time series, namely stationarity in the time series sense, is missing. Even if the time series were stationary in the time domain, successive wave heights are not independent. The luxury of an independent random sample is not available.

Most investigators of the statistical properties of wave records have not considered these complications. This approach has not been followed in this study. It will not be assumed that successive samples of N wave heights are from a stationary process and that within this sample the wave heights are independent.

The average wave height can be estimated in many different ways, implying the possibility of different values of $E_o^{\frac{1}{2}}$ and E_o. The first is from simply averaging all the wave heights and using equation (116). A second would be to use the variance of the wave record computed from the area under the estimated spectrum. The data in Tables 1, 2 and 3 provide a way to compute E_o, the area under the spectrum, from the tabulated values of RHS. This value can then be used to compute the mean wave height from (116).

A third way to estimate the average wave height is to find the maximum likelihood estimate of E_o. For an independent sample of size N, the probability of drawing that sample is

$$L(h_1,\ldots,h_n) = f(h_1,h_2,\ldots,h_n) = \prod_1^n \frac{h_i}{4\,E_o} \exp\,(-h_i^2/8E_o) \qquad (119)$$

The logarithm of this likelihood function is

$$\ln L(h_1,\ldots,h_n) = \sum_1^n \ln \frac{h_i}{4} - \ln E_o - h_i^2/8E_o \qquad (120)$$

This function is maximized by differentiating it with reference to E_o and setting the derivative equal to zero.

The result is the maximum likelihood estimate of E_{oMLE}.

$$E_{oMLE} = \frac{1}{8n} \sum_1^n h_i^2 \qquad (121)$$

This estimate of E_o would be unbiased if the sample were an independent random sample since

$$(E_{oMLE}) = \int_o^\infty \ldots \int_o^\infty \frac{1}{8n} \sum_1^n h_i^2 (\frac{h_i}{4E_o} \exp(-\, h_i^2/8E_o))dh_1,\ldots dh_n = E_o \qquad (122)$$

It follows that the best estimate of $\overline{h}$ may not be the average of the values of h, but the value computed from

$$\overline{h} = 2.5066\ (E_{oMLE})^{\frac{1}{2}} \qquad (123)$$

The fourth and last way that the mean wave height can be estimated is to find the best fitting value for E_o for the cdf over the full range of heights. The use of a limited range of heights as in Pierson and Salfi ((1982) (unpublished)) is misleading.

The sample of size N is first ordered from the lowest to the highest wave such that

$$h_1 < h_2 <\cdots<\cdots < h_{j-1} < h_j <\cdot<\cdot h_N \qquad (124)$$

The sample cdf is then given, as defined previously, by,

$$F(h_j) = \frac{j}{N} \text{ at } h = h_j \qquad (125)$$

The difference between it and the theoretical cdf is

$$D_j = \frac{j}{N} - (1 - \exp(- h_j^2/8E_o)) \qquad (126)$$

The best fit value of E_o is that value that will minimize

$$D = \sum_1^n D_j^2 \qquad (127)$$

A computer program has been developed that searches for that value of E_o that minimizes D.

The use of the significant wave height became popular before the availability of digital wave recording methods and spectral analysis. Often the one-third highest waves were determined by inspection of a graph of the waves on chart paper. The result was a considerable saving of time in data reduction.

From the properties of the cdf given previously, in a random sample of 200, the expected number of waves that exceed F(2/3) is 200/3, or $66.6\overline{6}$, or 67. The standard deviation is $(400/9)^{\frac{1}{2}}$, or (20/3), or $6.6\overline{6}$, or 7. The significant height, since one is not sure that one third of the waves in a random sample actually comprise all values belonging to the set of the one third highest waves in the population, need not be the average of just the one third highest. For an ordered sample this provides an estimate of the variability

$$\frac{1}{74} \sum_{127}^{200} h_i < h_{\frac{1}{3}} < \frac{1}{60} \sum_{141}^{200} h_i \qquad (128)$$

of this quantity.

A similar analysis for the one tenth highest waves yields 20 for the expected value and $(200(.1)(.9))^{\frac{1}{2}} \stackrel{\sim}{=} 4$ so that

$$\frac{1}{24} \sum_{177}^{200} h_i < h_{\frac{1}{10}} < \frac{1}{16} \sum_{185}^{200} h_i \qquad (129)$$

These equations do not consider that the sample may not be random. If the effective sample size were, say, 100, disguised at a sample of 200, the numbers to use would be 67 ± 9 and 20 ± 6 so as to broaden the area of uncertainty.

On the basis of the discussion of the sample cdf, it will usually happen that less than 20 waves from a sample of 200 will belong to the one tenth highest and that less than 67 will belong to the one third highest.

Each sample of 200 waves was processed in the same way so as to produce a table similar to Table 8, which is for records 49 and 50. All 55 tables are given in the appendix.

The top part of Table 8 for record 49 has 15 columns and 11 rows. The first 3 columns are the averages of the 24 highest, the 20 highest and the 16 highest waves as shown along the diagonal of the 10 by 10 array. Should 4 more heights actually belong to the one tenth highest, the value for the average of the 1/10th highest would be 37.0. Should exactly 20 heights belong to the one tenth highest, as is usually assumed, then the value would be 37.8. Finally should the sample only contain 16 waves, the value would be 39.3.

The next three rows are the corresponding estimates of the one third highest waves with values of 29.1, 29.9 and 30.8 on the diagonal. The value, 19.5 is the average of the 200 waves (HBAR). The maximum likelihood estimate (HMLE) is 19.7 for the mean. The value for the best fit cdf (HCDF) is 19.7 for the mean, and the mean value based on the area under the spectrum is 20.7 (HSPT).

The values for HMLE, HCDF and HSPT were computed from estimates of $(E_o)^{\frac{1}{2}}$, which is given in the column, E_o. The column labled E_o is the square root of E_o, which was computed to more significant figures before rounding and tabulation.

Each of the first ten columns in Table 8 represents a statistic that can be obtained from either the data in the form of h_1, h_2,....h_{200} or in the form of the estimated spectrum. From each statistic a statistic for $E_o^{1/2}$, (and E_o) can be obtained as some constant times the value, or the square of the value, that was obtained. From each of the possibly different values of $E_o^{1/2}$, the average of the one tenth highest, one third highest and of all the sample can be obtained. The columns thus show ten different ways to obtain the quantity headed by the column when appropriate nearly quantities are shifted to an appropriate column. As printed H10P can be shifted right, H10M left, HSIP right, HSIM left and HMLE, HCDF and HSPT left so as to form three columns under H10H, HSIG and HBAR.

Given a very large random sample, say two million, from a Rayleigh pdf, it would be anticipated that all of the values in each column would agree to about ± 0.4%. For a non-random sample of 200 waves, the values as computed by these different methods for H10H for record 49 are all, but one, within the range of uncertainty for that value of 37.0 to 39.3 ft. The exception is the value obtained from the spectral area which is 41.0 feet.

There are quite small differences between corresponding values in a given column and the values obtained from any one of the three values, HBAR, HMLE and HCDF, always with the exception of those obtained from the area under the spectrum. Both records 49 and 50 have these features. As a general statement, with perhaps an exception once in a while, the 55 data arrays have this feature.

The value across from H200 is the value of the highest wave in the sample. The values in the columns H5 is the height such that for 5% of the random samples of 200 waves with an $(E_o)^{1/2}$ value from the main diagonal (or as tabulated), the highest wave in the sample would be less than the tabulated value. Those in HMP and H200 are respectively the most probable and the expected values for the highest wave in 200. The value H95 is the value to be exceeded only 5% of the time. For record 49, the sample value of 46.5 lies between H5 and HMP for all values except the two from the spectrum. For record 50, the sample value of 65.4 is greater than H95 for all values except H10M, HSIM and HSPT.

TABLE 8 Statistics Obtained from Samples 49 and 50 as Defined in the Text.

105

49	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	37.0				29.1		18.2				7.3	42.2	47.4	50.0	59.1
H10H		37.9			29.7		18.6				7.4	43.1	48.5	51.2	60.4
H10M			39.3		30.0		19.3				7.7	44.8	50.3	53.1	62.7
HSIP		37.0		29.1			18.2				7.3	42.2	47.5	50.1	59.1
HSIG		38.0			29.0		18.7				7.5	43.3	48.7	51.4	60.7
HSIM		39.2				30.8	18.7				7.7	44.6	50.2	53.0	62.5
HBAR		38.2			30.0		18.8				7.5	43.5	48.9	51.6	60.9
HMLE		38.3			30.1			18.9			7.5	43.6	49.0	51.7	61.1
HCDF		38.5			30.3				19.0		7.6	43.9	49.4	52.1	61.5
HSPT		41.0			32.3					22.2	8.1	46.8	52.6	55.5	65.5
H200														46.5	

		H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM	HCDF	2.045	2.500	2.595	1.925	1.975	2.035	1.241	3.070
NORM	HBAR	2.470	2.525	2.421	1.944	1.995	2.055	1.253	3.101
NORM	HMLE	2.463	2.519	2.614	1.939	1.990	2.050	1.250	3.093

199

50	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	38.9				30.6		19.1				7.6	44.4	49.9	52.6	62.1
H10H		39.9			31.0		19.6				7.8	45.5	51.1	53.9	63.7
H10M			41.9		33.0		20.6				8.2	47.8	53.7	56.7	66.9
HSIP		38.9		30.6			19.2				7.6	44.4	49.9	52.7	62.2
HSIG		40.0			31.0		19.7				7.8	45.6	51.2	54.0	63.8
HSIM		41.1				32.3	19.7				8.1	46.9	52.7	55.6	65.6
HBAR		39.6			31.2		19.5				7.8	45.2	50.8	53.6	63.2
HMLE		40.1			31.6			19.7			7.9	45.8	51.4	54.3	64.1
HCDF		40.1			31.5				19.7		7.9	45.7	51.4	54.2	64.0
HSPT		42.0			33.0					20.7	8.2	47.9	53.8	56.8	67.1
H200														65.4	

		H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM	HCDF	2.472	2.534	2.663	1.945	1.996	2.053	1.238	4.155
NORM	HBAR	2.502	2.565	2.695	1.969	2.021	2.078	1.253	4.206
NORM	HMLE	2.469	2.531	2.660	1.943	1.994	2.051	1.237	4.151

If E_o were really known, and if all the other ifs involved were correct, the heights in a particular sample could be normalized by Eq. (111). Double the normalized value would then correspond to the corresponding heights for a spectrum with a value of E_o equal to one. Since only various estimates of the value of E_o are available, the statistics have been normalized in three different ways by using estimates based on HCDF, HBAR and HMLE so as to obtain normalized values of the quantities across the top in the eight by three array below the main array. Typically, the normalized values differ, for the two samples shown, in the second decimal place.

An examination of the tables for all 55 samples, as given in the appendix, raises a large number of interesting questions. One might be: Despite the indications in the data, is it possible that a combination of reasons could support the idea that heights computed from the area under the spectrum are the best values to use? Another could be: Have actual departures of samples of 200 consecutive waves been shown, at some confidence level, to differ from the Rayleigh pdf? An attempt to provide properly qualified answers to these questions, and others, will be made.

Figures 27, 28 and 29 are plots of the normalized values given on the bottom three rows of each of the tables in the appendix with the normalizations based, in order, on the best fit cdf, the mean, and the maximum likelihood estimate. For each record number from the bottom to the top, the value are the normalized values of HBAR, HSIP, HSIG, HSIM, H10P, H10H, H10M and H200. All three figures, at first glance, are remarkably similar except that the mean normalized by the mean is a constant. (The appropriate constant was rounded to 1.253, and calculations faithfully discovered that the standard deviation was 0.0003).

In general, the figures show that the smaller the sample; i.e. 200, 67±7, 20±4, and 1, the larger the variability of the estimate about its expected value as shown by the arrow on the right hand side. The horizontal bars are analogous to confidence intervals, but, being based on the incorrect assumption of a random sample of 200 values, they are not broad enough to enclose the expected values often enough.

Table 9 shows the distribution of the normalized mean for sample intervals

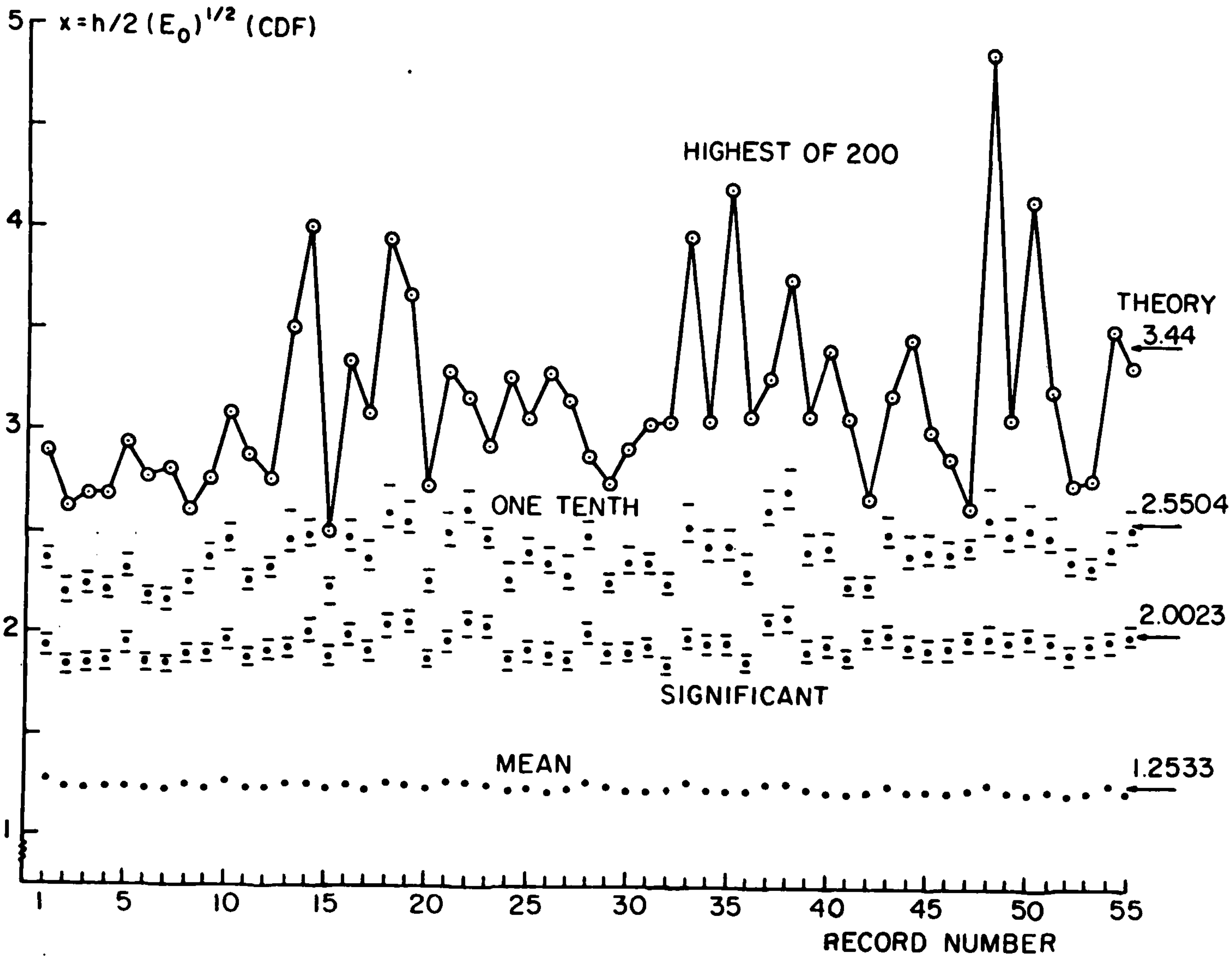

FIGURE 27 Values of $\bar{H}$, HSIG, H10 and H200 Normalized by the Best Fit CDF.

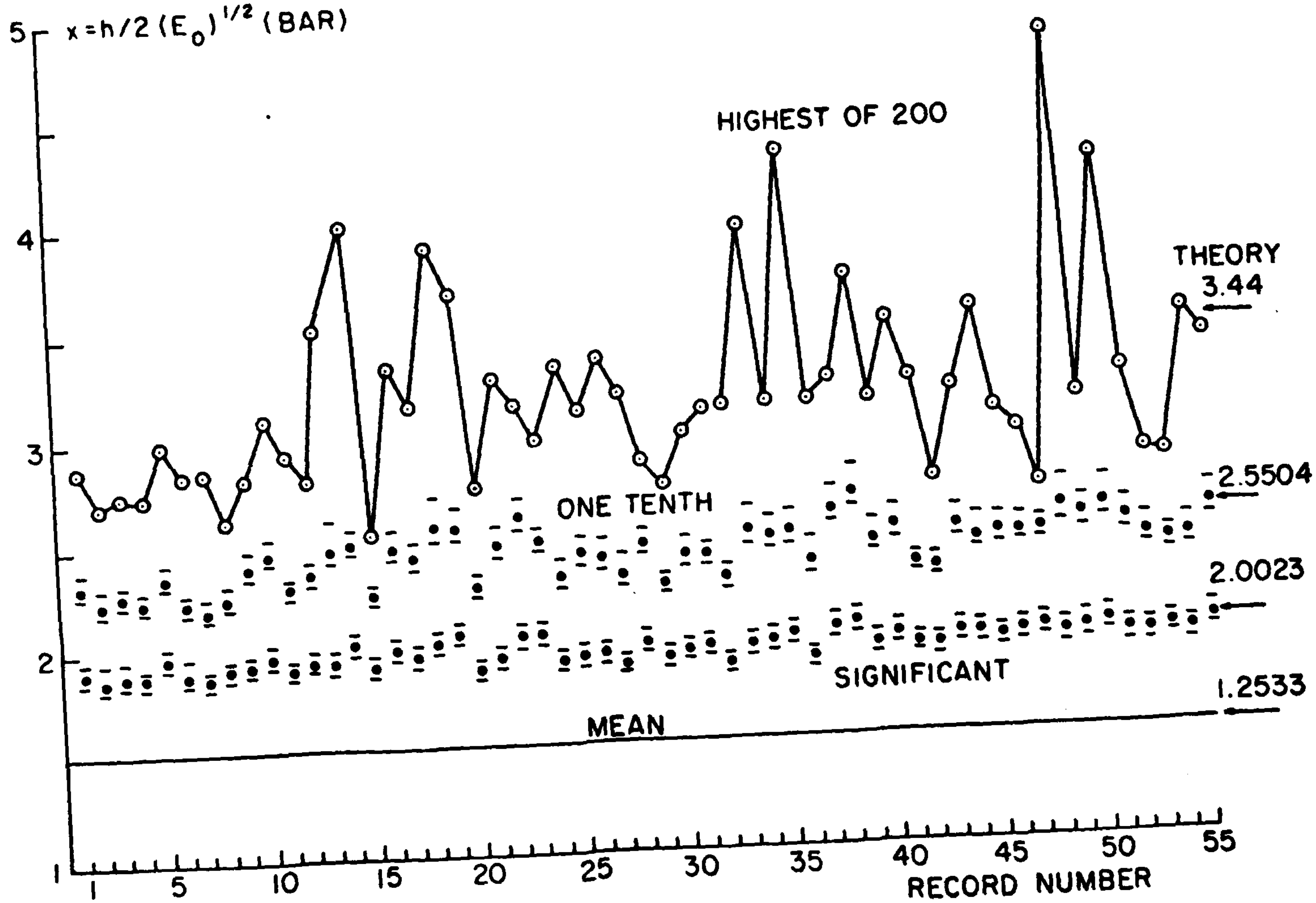

FIGURE 28 Values of $\bar{H}$, HSIG, H10 and H200 Normalized by the Mean.

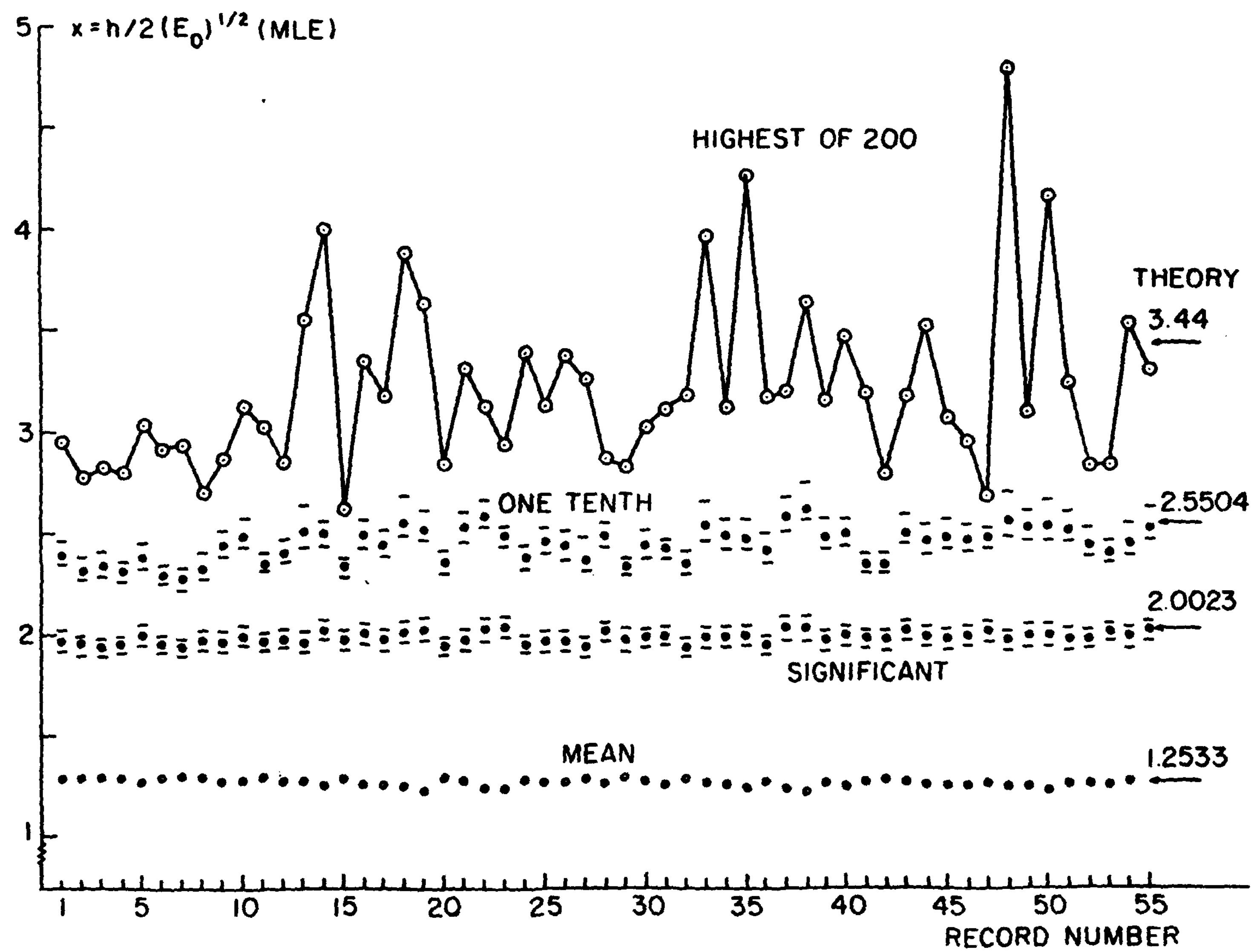

FIGURE 29 Values of $\tilde{H}$, HSIG, H10 and H200 Normalized by the Maximum Likelihood Estimate.

TABLE 9 Counts per Class Interval for the Normalized Mean. Theoretical Value Equals 1.2533.

	$\overline{H}$ NORMALIZED BY		
	CDF	MEAN	MLE
INT			
1.18			
1.20	1		
1.22	24		4
1.24	19	55	17
1.26	11		21
1.28			13
1.30			

TABLE 10 Counts per Class Interval for Various Estimates of the Significant Wave Height. Theoretical Value Equals 2.0023.

	NORMALIZED BY								
	CDF			MEAN			MLE		
	HSIP	HSIG	HSIM	HSIP	HSIG	HSIM	HSIP	HSIG	HSIM
INT									
1.76									
1.78	1								
1.80	3								
1.82	5	2		2					
1.84	7	2		7					
1.86	7	6	2	6	3				
1.88	4	5	2	7	5		6		
1.90	9	6	4	8	6	2	13		
1.92	4	6	4	8	5	6	17	6	
1.94	6	7	6	9	8	5	13	8	
1.96	3	6	6	3	8	4	5	16	6
1.98	3	5	6	2	9	7	1	13	4
2.00	2	3	6		4	5		7	10
2.02	1	2	4		3	8		5	14
2.04		3	5		3	8			12
2.06		1	4		1	4			4
2.08			2			2			5
2.10		1	2			3			
2.12			1			1			
2.14									
2.16			1						
2.18									

TABLE 11 Counts per Class Interval for Various Estimates of the Average of the One Tenth Highest. Theoretical Value Equals 2.5504.

	NORMALIZED BY								
	CDF			MEAN			MLE		
	H10P	H10H	H10M	H10P	H10H	H10M	H10P	H10H	H10M
INT									
2.05									
2.10	2								
2.15	5	3		4	1				
2.20	8	5	2	7	4		1		
2.25	3	7	5	5	7	3	7	3	
2.30	8	5	6	6	5	8	11	10	2
2.35	9	9	3	8	7	3	9	7	6
2.40	10	8	7	16	9	5	16	9	6
2.45	5	8	6	5	12	7	8	14	6
2.50	2	5	9	3	4	10	2	9	12
2.55	2	2	5	1	4	7	1	2	10
2.60	1	2	4		1	4		1	7
2.65			3		1	6			4
2.70		1	4			1			2
2.75						1			
2.80			1						
2.85									

of 0.02 with the lower end of the class interval tabulated and the upper end 0.01999 greater. The CDF normalizes low and the MLE normalizes high compared to the expected value of 1.2533.

Table 10 shows the distribution of the various estimates of the normalized significant height. It is notable that the estimates normalized by the MLE and HSIM come the closest to scattering uniformly about the expected value of 2.0023. The maximum likelihood estimate of $(E_0)^{\frac{1}{2}}$ is generally smaller than the one based on the mean. It is smaller because high values of the h_i are usually missing from the sample as described in the section on sample cdf's. Normalization then makes the other estimates larger. When combined with a too small correction for the missing high values, the net effect is to obtain a nearly correct value.

Table 11 shows the distribution of the various estimates of the average of the one tenth highest. The class intervals are 0.05 instead of 0.02. The spread is much greater, but again the value of H10M comes the closest to scattering the least about the value of 2.5504.

Table 12 shows the distribution of the normalized value of the highest wave in each sample. Two class intervals are tabulated under each interval heading to save space. For example for CDF at 2.8, there was one value between 2.8 and 2.84999 and these were four values between 2.85 and 2.8999. The very large scatter of the normalized values is better judged from Figs. 27, 28 and 29. Most of the data are below the value of 3.44.

Table 13 shows the means minus and plus one standard deviation for each of the eight quantities for the normalized values from the 55 samples. Underlined pairs show that the theoretical expected value lies within the range shown. The results are exactly what one would anticipate from the preceding analyses and are more easily interpreted in terms of Table 14 which expresses all of the values in terms of percentages of the theoretical values. For $\overline{H}$, computed two ways other than from $\overline{H}$ itself, one methods is 0.7% too low but is within plus one standard deviation. The other is 1% too high but is within minus one standard deviation.

The normalized significant height averages to 2.9%, 2.2% and 1.2% less than the expected value, but the range of the average plus one standard

TABLE 12 Counts per Class Interval (See Text) For Normalized Values of the Highest Wave in Each Sample. The Expected Value is 3.441

	NORMALIZED BY		
INT	CDF	MEAN	MLE
2.4			
2.5	1,0	1,0	
2.6	3,3	1.2	1.2
2.7	2,5	4,2	0,3
2.8	1,4	5,2	6,2
2.9	3,0	2,3	4,1
3.0	2,9	1,4	3,2
3.1	1,2	6,3	6,6
3.2	1,4	2,1	2,1
3.3	2,0	3,1	2,2
3.4	1,1	0,2	0,1
3.5	2,0	2,0	3,0
3.6	0,1	0,2	2,0
3.7	1,0		
3.8	0,0	0,1	0,1
3.9	1,2	1,1	1,1
4.0	0,0		
4.1	0,2		0,1
4.2		1,1	0,1
4.3			
4.4			
4.5			
4.6			
4.7			0,1
4.8	0,1	1,0	
4.9			
5.0			

deviation encloses the theoretical value for all three normalizations. Averaging the sixty highest produces an average within 0.4%, 0.3% and 1.3% of the expected value. The theoretical value is on the high side when normalized by the CDF and on the low side when normalized by both the mean, and the MLE, but still within one standard deviation.

The normalized average of the one tenth highest waves is low and adding one standard deviation does not enclose the theoretical value. The average of the sixteen highest yields average normalized values that are low by 2.7% 2.0% and 1.1% and lie within plus one standard deviations of the expected value.

The average of the normalized values of the highest wave in each sample is too low by 8.8%, 7.7% and 6.9%,but plus one standard deviation encloses the expected value. The distribution of these values has many strange features, to be discussed later.

TABLE 13 NORMALIZED WAVE STATISTICS SHOWING THE MEAN MINUS ONE STANDARD DEVIATION, THE MEAN AND THE MEAN PLUS ONE STANDARD DEVIATION.

		NORMALIZED BY		
	THEORY	HCDF	$\overline{H}$	HMLE
$\overline{H}$	1.2533	1.230 < <u>1.244</u> < <u>1.258</u>	1.253	<u>1.249</u> < <u>1.266</u> < 1.283
HSIG+	2.0023	1.840 < 1.898 < 1.957	1.865 < 1.912 < 1.959	1.906 < 1.930 < 1.953
HSIG	2.0023	1.881 < <u>1.945</u> < <u>2.0086</u>	1.909 < <u>1.959</u> < <u>2.0108</u>	1.950 < <u>1.978</u> < <u>2.006</u>
HSIG-	2.0023	1.926 < <u>1.995</u> < <u>2.0641</u>	<u>1.952</u> < <u>2.009</u> < 2.0655	<u>1.997</u> < <u>2.029</u> < 2.061
H10+	2.5504	2.227 < 2.347 < 2.467	2.259 < 2.364 < 2.4688	2.310 < 2.386 < 2.462
H10	2.5504	2.265 < 2.393 < 2.521	2.297 < 2.409 < 2.521	2.348 < 2.432 < 2.516
H10-	2.5504	2.335 < <u>2.482</u> < <u>2.629</u>	2.368 < <u>2.499</u> < <u>2.630</u>	2.420 < <u>2.522</u> < <u>2.624</u>
H200	3.44	2.687 < <u>3.155</u> < <u>3.623</u>	2.721 < <u>3.176</u> < <u>3.63</u>	2.773 < <u>3.204</u> < <u>3.635</u>

TABLE 14 NORMALIZED WAVE STATISTICS AS PERCENT OF THEORY FROM TABLE 13.

		NORMALIZED BY		
	THEORY	HCDF	$\overline{H}$	HMLE
$\overline{H}$	1.2533	98.1 < 99.3 < 100.4	100	99.7 < 101.0 < 102.4
HSIG+	2.0023	91.9 < 94.8 < 97.7	93.1 < 95.5 < 97.8	95.2 < 96.4 < 97.5
HSIG	2.0023	93.9 < 97.1 < 100.3	95.3 < 97.8 < 100.4	97.4 < 98.8 < 100.2
HSIG-	2.0023	96.2 < 99.6 < 103.1	97.5 < 100.3 < 103.2	99.7 < 101.3 < 102.9
H10+	2.5504	87.3 < 92.0 < 96.7	88.6 < 92.7 < 96.8	90.2 < 93.6 < 96.5
H10	2.5504	88.8 < 93.8 < 98.8	90.1 < 94.5 < 98.8	92.1 < 95.4 < 98.6
H10-	2.5504	91.5 < 97.3 < 103.1	92.8 < 98.0 < 103.1	94.9 < 98.9 < 102.9
H200	3.44	78.1 < 91.2 < 105.3	79.1 < 92.3 < 105.5	80.6 < 93.1 < 105.7

DOES E_o FROM THE SPECTRUM PREDICT THE SIGNIFICANT HEIGHT?

Figures 1 and 2 and Tables 1, 2, 3, and 8, along with the other 54 tables in the appendix, plus the theories appropriate to Figures 17 and 18, plus numerous references, plus the lack of a random sample and the peculiarities of the sample cdf, all seem to suggest that the significant wave height is not correctly given by either eqn. (3) (or eqn.(118)) if the area under the wave spectrum is used to find E_o.

The theories that support this concept are all variations of a linear underlying model and cannot describe the effects of higher order nonlinearities that could increase the heights of the higher waves in a time history so that the effects treated in the present theories would be cancelled out, either in part, completely or overcompensated for. Most wave recording methods at present use a digitized sample, obtained every Δt seconds, with 2^n values, find the estimate of the spectrum, truncate it at some upper frequency, find the area under the truncated spectrum and compute the significant wave height.

This procedure, apart from problems in the frequency response of the wave recorder, can introduce an additional contribution to the variance of the time history because of digitization noise that may or may not be white.

If for example,

$$\eta(o),\quad \eta(\Delta t),\ \eta(2\Delta t)\cdots\cdots\ \eta((\eta-1)\Delta t)$$

is the record that ought to have been digitized, and if each value has an error such that

$$\eta(o) + \varepsilon(o),\ \eta(\Delta t) + \varepsilon(\Delta t),\ \eta(2\Delta t) + \varepsilon(2\Delta t),\cdots\cdots$$

$$\eta((n-1)\Delta t) + \varepsilon((n-1)\Delta t),$$

then

$\mathrm{VAR}(\eta(t) + \varepsilon(t))$ is estimated by

$$\mathrm{VAR} = \frac{1}{n}\sum_{o}^{n-1} (\eta(p\Delta t) + \varepsilon(p\Delta t))^2$$

$$= \frac{1}{n}\sum_{i}^{n-1}\left\{(\eta(p\Delta t))^2 + 2\eta(p\Delta t)\ \varepsilon(p\Delta t) + (\varepsilon(p\Delta t))^2\right\} \qquad (130)$$

The second term in eqn. (130) may average to zero, but the last one cannot. The errors may even have a spectrum that is not simply more or less a constant

at all frequencies below $(2\Delta t)^{-1}$. The variance found from a particular time history can then be too high, and when the significant height is computed from this variance, it is too high, for one more reason other than all of the others found so far, compared to the average of the one third highest waves in the same sample.

Given only a sample of crest to trough wave heights, plus the fact that successive values are not independent, plus the particiuliarities of sample cdf's from a Rayleigh-like population, there may be no optimum way either to obtain the best fit value for the single Rayleigh parameter or to find out which of many possibilities may modify the Rayleigh pdf to some other pdf with additional parameters that result in forms that have a limiting shape of the Rayleigh.

The best fit cdf, the mean, and the maximum likelihood estimate for E_o (or $E_0^{\frac{1}{2}}$) give slightly different results that are all below the value from the spectra of the waves from Camille. The lack of high waves in the sample of limited effective size not only affects the direct estimates of the significant wave height and the average of the one tenth highest but also affects the estimates that involve the entire sample.

Figure 30 shows schematically the kind of thing that can happen for a sample from a Rayleigh pdf. The solid curve is the normalized pdf and the dashes represent a smoothed version of a possible sample. The estimate of the mean is shown schematically in Fig. 31. In Fig. 30 two thirds of the area under the theoretical pdf lies to the left of $(2 \ln 3)^{\frac{1}{2}}$, seven thirtieths lies between $(2 \ln 3)^{\frac{1}{2}}$ and $(2 \ln 10)^{\frac{1}{2}}$ and one tenth lies above $(2 \ln 10)^{\frac{1}{2}}$. The contribution to the mean for values less than $(2 \ln 3)^{\frac{1}{2}}$ can be too large compared to what ought to have been found, and the contribution for values greater than $(2 \ln 10)^{\frac{1}{2}}$ could be too small as indicated in Fig. 31.

Consider the values that contribute to the normalized mean as in equation (131),

$$\frac{\bar{h}}{2(E_o)^{\frac{1}{2}}} = \int_o^{(2 \ln 3)^{\frac{1}{2}}} x^2 e^{-x2/2} dx + \int_{(2 \ln 3)^{\frac{1}{2}}}^{(2 \ln 10)^{\frac{1}{2}}} x^2 e^{-x^2/2} dx + \int_{(2 \ln 10)^{\frac{1}{2}}}^{\infty} x^2 e^{-x^2/2} dx \quad . \quad (131)$$

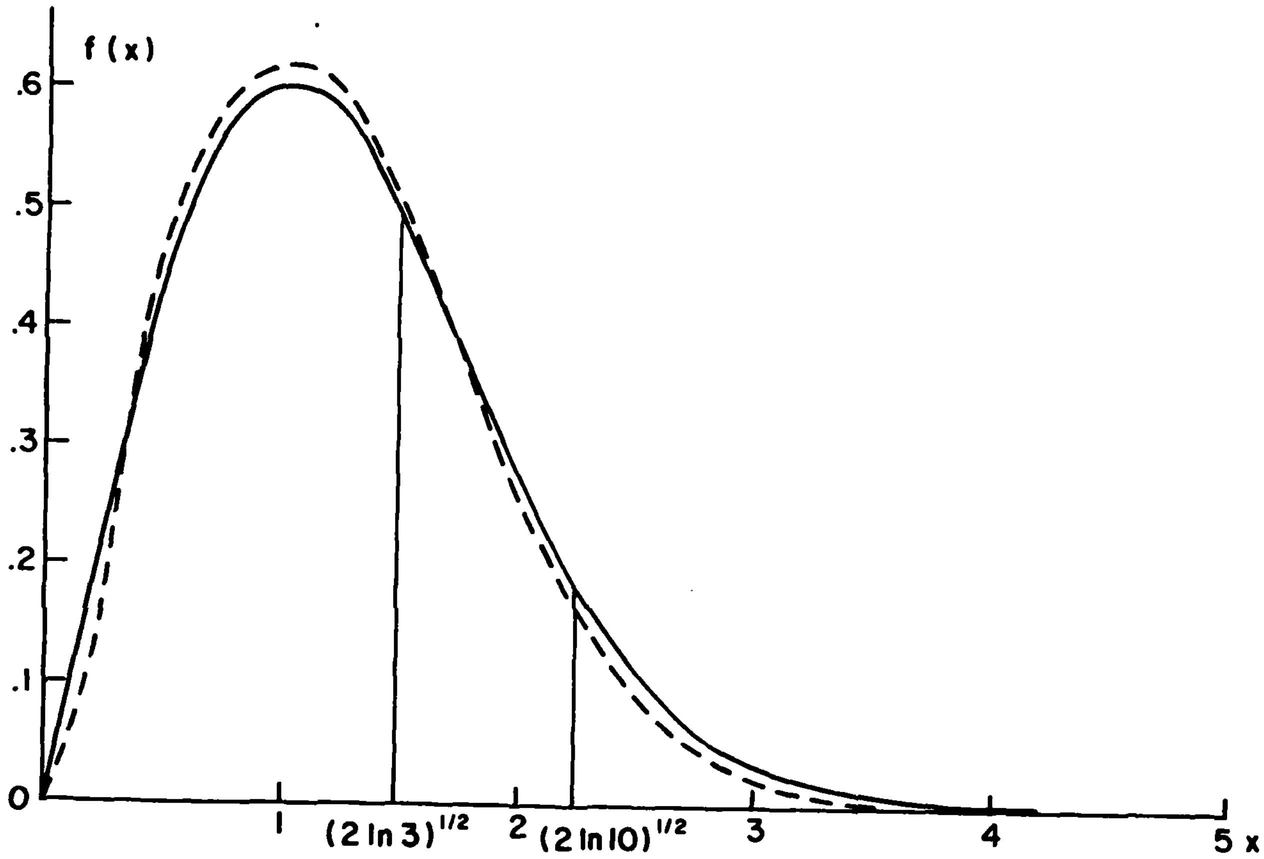

FIGURE 30 Distortion of a Typical Rayleigh pdf(-) by a Typical Small Sample (---).

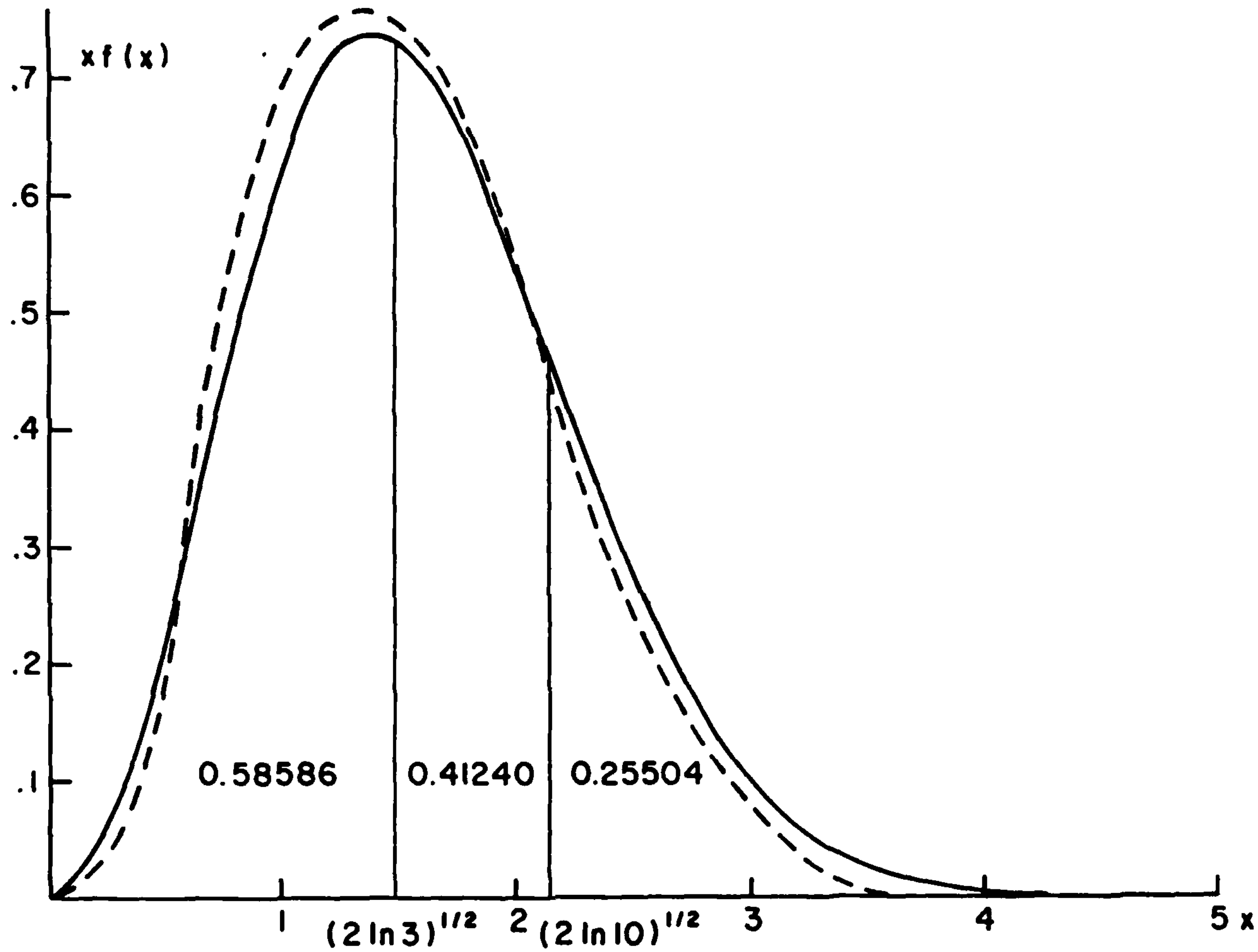

FIGURE 31 The Effect of this Distortion on the First Moment.

which upon integration yields,

$$\frac{\bar{h}}{2(E_o)^{\frac{1}{2}}} = 0.58586 + 0.41240 + 0.25504 \tag{132}$$

If the sample is such that it does not contain enough high waves, then there must be an excess of lower waves. For an actual sample, the contribution to the mean from the lowest 2 n/3 waves and from the highest n/10 waves ought to have a ratio of (0.58586)/(0.25504) = 2.29713. The appropriate sums can be

$$\bar{h} = \frac{1}{200}\sum_{1}^{133} h_i + \frac{1}{200}\sum_{134}^{180} h_i + \frac{1}{200}\sum_{181}^{200} h_i \tag{133}$$

found with some ingenuity from the values in Table 8, and the corresponding tables for the total of 55 samples in the appendix. The column for H10H provides additional significant figures for the calculation.

Table 15 shows the results for the 55 samples tabulated according to sample number. The first value is the value from the spectrum. The second is the value from the highest 20 waves. The third, R_1, is the value of the ratio of the first term in (133) to the value of the last term in (133). For most samples, this ratio exceeds the value of 2.29713. The ratio of the sample ratio to the theoretical ratio is shown in the column, R_n. This ratio times the value estimated from the one tenth highest waves yields a corrected value for the Rayleigh parameter. The difference before correction is shown by D_1 and after correction by D_2. If H10M is used instead of H10H and the calculation repeated (with the value treated as if it were the average of the 20 highest waves) the final difference is given by D_3.

TABLE 15 Possible Effect of Sample Size on the Estimation of the Rayleigh Parameter

	$(EOSP)^{1/2}$	$(E10)^{1/2}$	R_1	R_n	$(E10C)^{1/2}$	D_1	D_2	D_3
1	0.902	0.765	2.636	1.148	0.877	0.14	0.02	0.00
2	0.902	0.725	2.869	1.249	0.906	0.17	-0.00	-0.05
3	0.902	0.745	2.705	1.178	0.877	0.15	0.02	0.00
4	1.039	0.863	2.865	1.247	1.076	0.18	-0.04	-0.06
5	1.078	0.852	2.504	1.090	0.929	0.23	0.15	0.10
6	1.118	0.902	2.812	1.224	1.104	0.22	0.02	-0.01
7	1.098	0.882	2.874	1.251	1.104	0.22	-0.01	-0.05
8	1.235	1.019	2.742	1.194	1.217	0.22	0.02	-0.02
9	1.235	1.078	2.593	1.129	1.217	0.16	0.02	-0.02
10	1.333	1.196	2.391	1.041	1.245	0.13	0.09	0.08
11	1.666	1.372	2.748	1.196	1.642	0.30	0.03	-0.02
12	1.686	1.353	2.595	1.130	1.528	0.34	0.16	0.12
13	1.725	1.510	2.455	1.069	1.613	0.21	0.12	0.02
14	1.843	1.627	2.355	1.025	1.669	0.21	0.17	0.11
15	2.059	1.588	2.702	1.176	1.868	0.47	0.17	0.15
16	1.961	1.725	2.411	1.050	1.811	0.23	0.15	0.09
17	2.059	1.745	2.458	1.070	1.867	0.32	0.19	0.13
18	2.117	1.902	2.221	0.967	1.838	0.22	0.28	0.18
19	2.333	2.000	2.209	0.962	1.923	0.33	0.41	0.33
20	2.411	2.117	2.795	1.217	2.576	0.29	-0.15	-0.24
21	2.509	2.294	2.409	1.049	2.405	0.22	0.10	0.00
22	2.706	2.490	2.152	0.937	2.333	0.22	0.38	0.30
23	2.784	2.451	2.253	0.981	2.403	0.33	0.38	0.30
24	2.686	2.313	2.239	0.975	2.254	0.38	0.43	0.36
25	2.666	2.215	2.465	1.073	2.377	0.45	0.29	0.21
26	2.745	2.333	2.479	1.079	2.518	0.41	0.22	0.12
27	2.823	2.490	2.690	1.171	2.915	0.33	-0.10	-0.23
28	2.980	2.627	2.424	1.055	2.772	0.35	0.21	0.11
29	3.058	2.588	2.663	1.159	3.000	0.47	0.06	-0.01
30	2.960	2.509	2.513	1.094	2.745	0.45	0.21	0.11
31	3.176	2.725	2.480	1.080	2.942	0.45	0.24	0.15
32	3.215	2.804	2.411	1.049	2.942	0.42	0.28	0.16
33	3.333	3.039	2.396	1.043	3.169	0.29	0.16	0.00
34	3.196	2.862	2.384	1.038	2.970	0.30	0.23	0.11
35	3.411	2.980	2.351	1.025	3.055	0.43	0.35	0.23
36	3.548	3.137	2.193	0.955	2.994	0.41	0.56	0.44
37	3.686	3.529	2.191	0.954	3.365	0.16	0.31	0.17
38	3.921	3.627	2.095	0.912	3.308	0.29	0.61	0.45
39	4.097	3.607	2.432	1.059	3.820	0.49	0.26	-0.13
40	4.117	3.646	2.370	1.032	3.762	0.47	0.36	-0.22
41	4.391	3.745	2.604	1.133	4.244	0.65	0.15	-0.03
42	4.803	4.215	2.622	1.141	4.811	0.58	-0.01	-0.12
43	4.940	4.490	2.374	1.033	4.639	0.45	0.30	-0.12
44	5.740	5.038	2.399	1.044	5.262	0.70	0.48	-0.25
45	5.842	5.313	2.447	1.065	5.658	0.53	0.18	0.03
46	6.509	5.979	2.391	1.041	6.223	0.53	0.29	-0.06
47	7.430	6.724	2.367	1.031	6.930	0.69	0.50	-0.30
48	7.999	7.528	2.314	1.007	7.582	0.47	0.42	0.04
49	8.038	7.411	2.324	1.012	7.497	0.62	0.54	-0.21
50	8.234	7.822	2.251	0.980	7.665	0.41	0.56	-0.18

Continued on page 166.

51	9.077	8.959	2.372	1.033	9.251	0.12	-0.17	-0.56
52	8.724	7.705	2.434	1.060	8.163	1.02	0.56	0.24
53	9.312	8.391	2.455	1.069	8.967	0.92	0.34	-0.09
54	9.940	9.293	2.469	1.075	9.988	0.65	-0.05	-0.45
55	11.018	10.430	2.242	0.976	10.179	0.59	0.84	0.38

The value of R_n is greater than one for 45 out of 55 samples. The corrected value of $(E10C)^{\frac{1}{2}}$ exceeds the value from the spectrum eight times. However, the averages of the ratios of $(E10C)^{\frac{1}{2}}$ to $(EOSP)^{\frac{1}{2}}$ is only 0.94 so that heights found from the spectrum are still high compared to heights from the crest to trough height samples. The values for D_3 show that corrections based on H10M may overcompensate for the effects considered since the corrected value exceeds or essentially equals the spectral value 27 times.

The one standard deviation bounds on HSIG+, HSIG, H10+, H10 and H10- in Table 14 all enclose 94% so that it may be asking too much to have a particular sample of significant wave heights verify within plus or minus 10% when the spectral variance is used. There is no convenient way to study the hypothesis that digitization errors account for the differences that are observed with the presently available data. The complete wave recording system would have to be analysed step by step.

It is difficult enough to record waves 72 feet, 65 feet, 69 feet and 70 feet from crest to trough without quibbling about a few feet of possible error in the measurements. Most other wave measuring system have far greater difficulties than the system used to measure these waves in Camille.

The choice that needs to be made is whether or not to use the significant height based either on HBAR or HMLE or on the spectral moment. The largest difference is 2.7 feet out of 35 feet (from the spectrum) for record 52 so that the significant height from the spectrum is 8% higher compared to that from the MLE. It seems the better part of wisdom until more is known about the vagaries of a particular wave recording instrument system, to use the spectral moment with the reservation that the result may be slightly too high for a particular sample.

If F(x) is the cdf appropriate to a random sample of size N, the cdf of the largest value is given by equation (134)

$$F_E(x) = (1 - F(x))^N \tag{134}$$

The pdf of the highest value in the sample is given by

$$f_E(x) = N(1 - F(x))^{N-1} \, d\, F(x)/dx \tag{135}$$

For the normalized Rayleigh distribution, these equations become

$$F_E(x) = (1 - e^{-x^2/2})^N \tag{136}$$

and

$$f_E(x) = N(1 - e^{-x^2/2})^{N-1} \, xe^{-x^2/2} \tag{137}$$

Figures 32, 33 and 34 show the normalized sample cdf's based on the best fit cdf, the mean, and the maximum likelihood estimate. Also shown are the graphs of equation (136) for N equal to 200, 100 and 50. The three theoretical curves nearly parallel each other and the distances at p = 0.5 separating the curves for 50 and 100 and for 100 and 200 are almost the same. This is, of course, a feature of the extreme value theory of Longuit-Higgins (1952) where the various properties of the extreme values vary as the logarithm of the sample size. Doubling the sample size to 200 from 100 only increases the expected value of the highest wave in the sample by about 15%.

The sample cdf's, based on 55 normalized extreme values, have little, if any, relation to the theory of extreme values in a random sample of 200 waves. Even the concept of an equivalent sample size, so useful in the interpretation of spectral estimates, leads to difficulties since this quantity is not necessarily constant from one sample to another. The three sample cdf's shift toward higher values depending on which of the three ways the normalizing parameter is chosen with normalization by the maximum likelihood estimate being the farthest to the right. This shows that overall, this parameter is smaller than the one found from the mean. The higher waves, some of which are missing from a particular sample, cause this parameter to

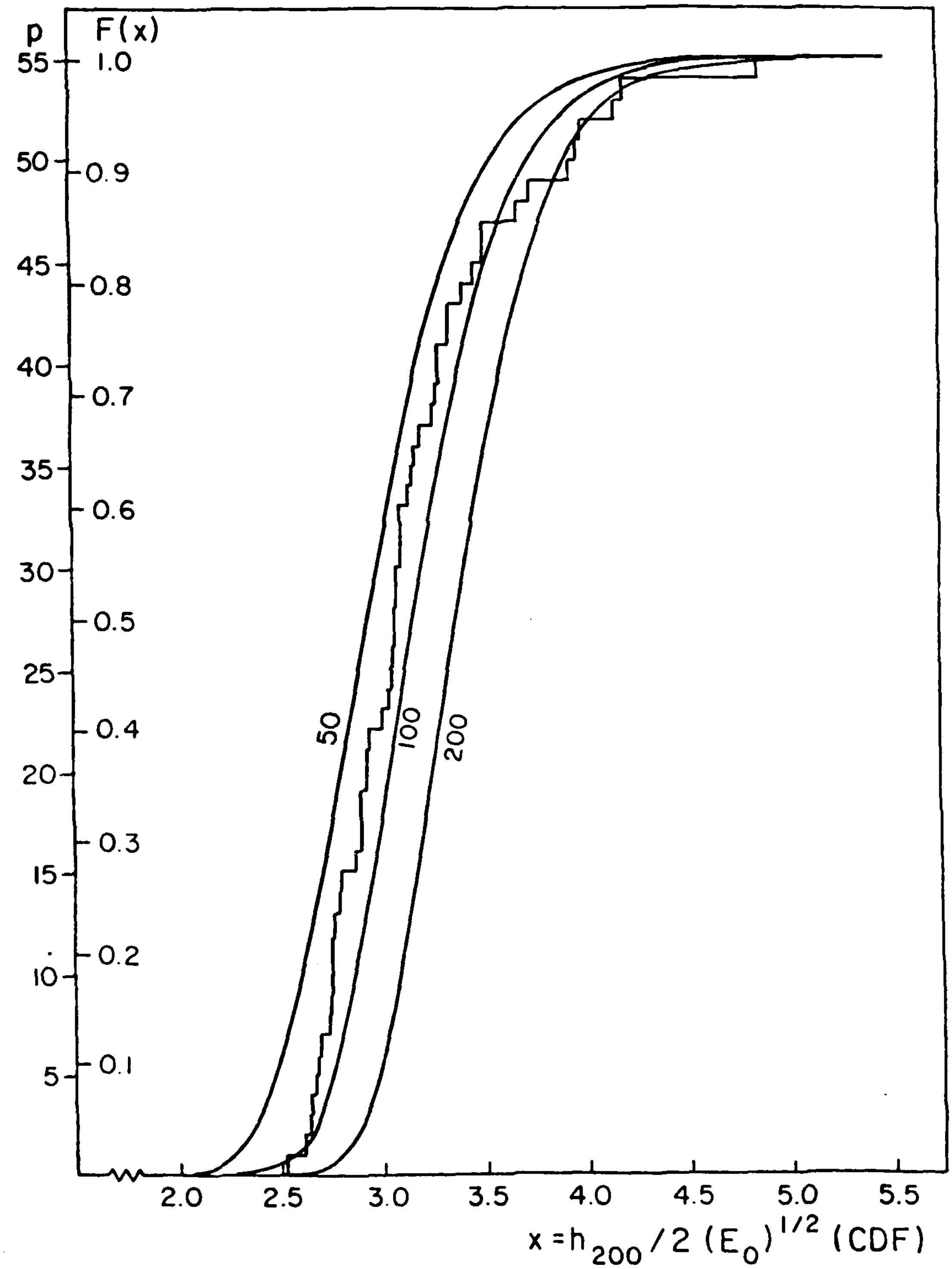

FIG. 32 Sample Extreme Value cdf and Theoretical Curves Normalized by the Best Fit cdf of each Sample.

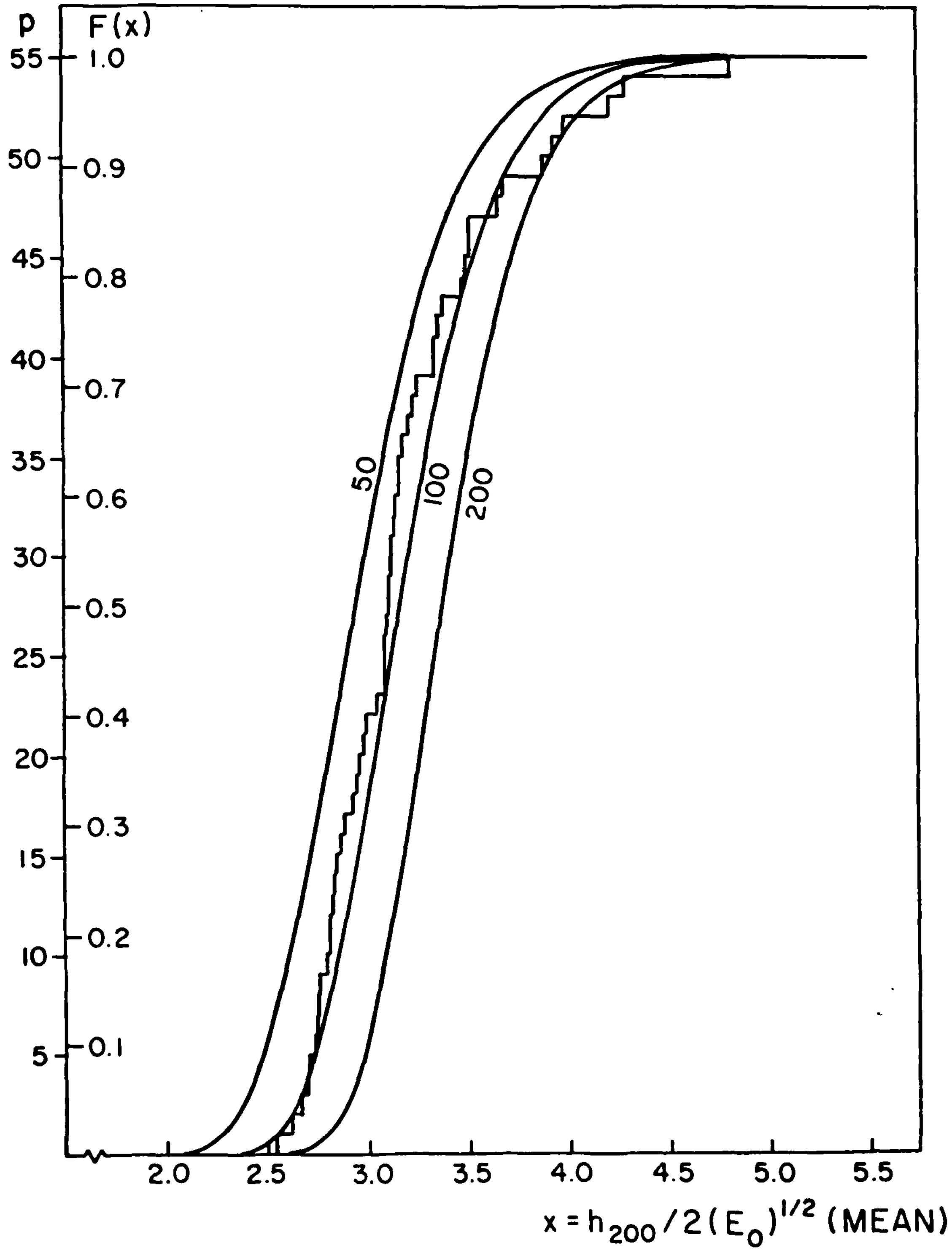

FIG. 33 Sample Extreme Value cdf and Theoretical Curves Normalized by the Mean of each Sample.

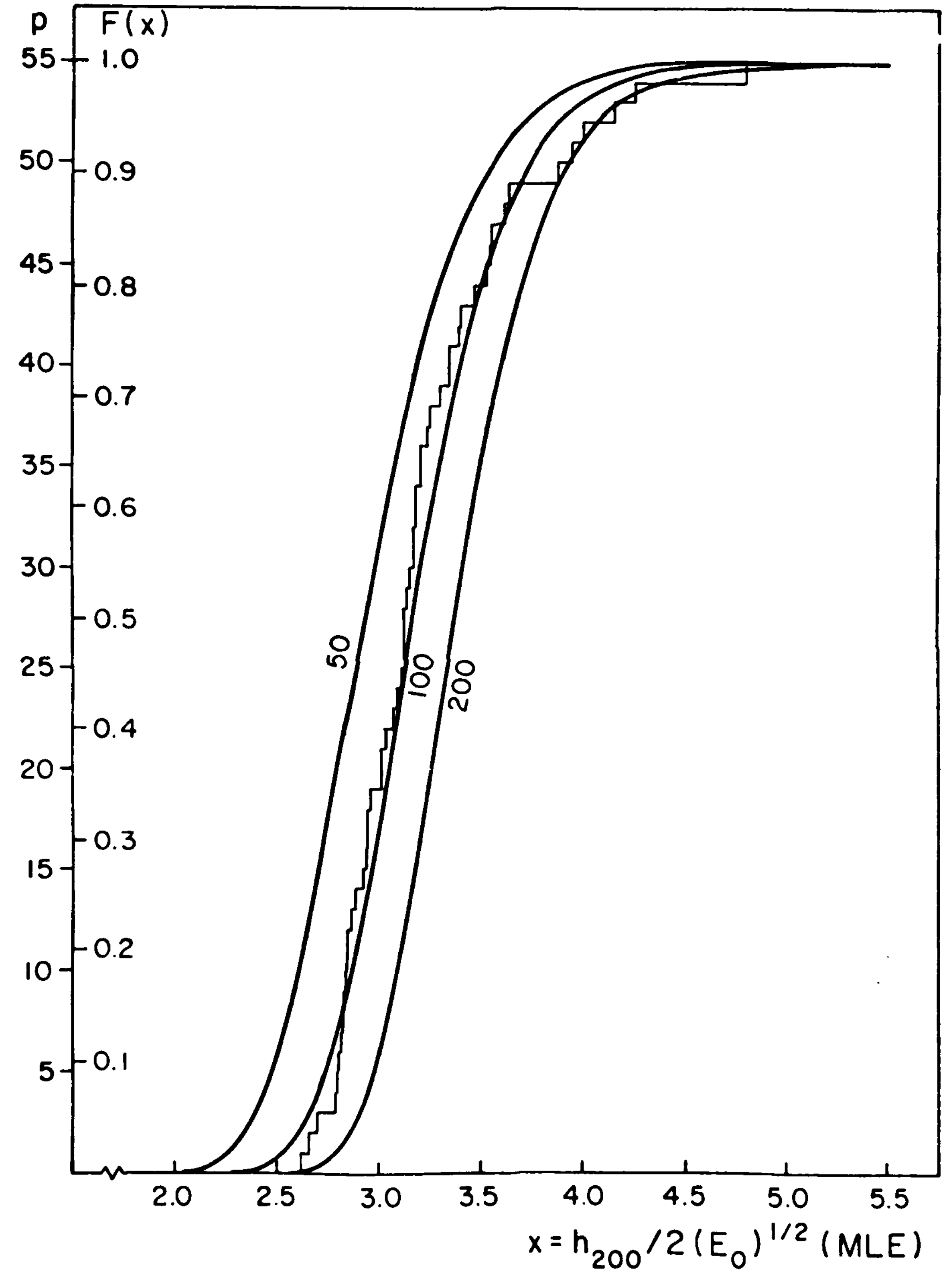

FIG. 34 Sample Extreme Value cdf and Theoretical Curves Normalized by the MLE of each Sample.

perhaps be underestimated because they have a greater effect on the estimate. The relative concentration of the normalized values relative to the MLE as in Tables 10 and 11 is also explained by this effect since this choice of parameter emphasises the higher waves in a sample.

Both Figures 33 and 34 suggest that the effective sample size may be somewhere between 80 and 100 for p up to 49. However, the six highest values for normalized H200 for records 14, 18, 33, 35, 48 and 50 switch from values near the theoretical curve for 100 to the theoretical curve for 200. The 11% highest waves in repeated samples of 200, that is the extreme of the extremes, are predicted quite well by the theory. Of these six extremes, the first three are close to the value for H95. For record 35 only the theoretical value from the spectral variance exceeds the observed height. For record 48, all theoretical values are less than the observed height. For record 50, all but three are less than the observed height.

For record 48, if the spectral parameter is used, the chance of exceeding a height of 72.2 feet is 0.00784 for a random sample of 200 waves. For 55 random samples of 200 waves the chance of a wave this large or larger is one or more of the samples is 0.351 so that this wave is not unusual if the extreme value theory is applied to a random sample of 200. If the sample size were only 100, the chance of finding a wave this high or higher in 55 sample would be 0.195, which is not too improbable.*

Based on the peculiar properties of sample cdf's for the individual samples, and on the lack of a random sample, there is no convincing evidence that the higher waves in a Rayleigh pdf are eliminated by breaking and that the pdf is modified in some way to account for breaking. The heights that can be attained by irregular trains of waves of different heights are not well understood, and the upper limit on how high they can get may not be discernable in typical samples.

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* For a moment based on HCDF, which is probably too low; the corresponding values are 0.00145, 0.08 and 0.04, a modestly rare occurrence.

A REVISIT OF THE RESULTS OF SPRING

Figures 1, 2 and 3 of this present study and Tables 1, 2 and 3 on pages 10 to 20 showed only the properties of the initial stages of the analysis of this same data by Spring (1978). It is now possible in view of the various points raised in the preceding sections to return to his study, reinterpret the results that were obtained and suggest possible alternative conclusions that might be drawn from his analysis.

In describing how tests were made to see whether or not the samples of crest to trough wave heights would fit the Rayleigh pdf, the following material is quoted directly from Spring with changes in equation numbers and figure numbers.*

"Borgman (1973) describes a method, attributed to D. Lee Harris of the Coastal Engineering Research Center, to test the appropriateness of a distribution function to ranked data. The method has been used in this study to test the Rayleigh, Forristall and a mean height distribution and is described below.

"According to Hogg and Craig (1970, page 13), an estimate of the distribution function from ranked data may be obtained from

$$P[h_{(r)}] = \frac{N-r+1}{N+1} \tag{138}$$

where $h_{(r)}$ is the height of the rth largest observed wave, N is the total sample size and r is the wave rank after the observed waves are sorted into descending heights. For the Rayleigh distribution

$$P[h_{(r)}] = 1-e^{-h^2/a^2} \tag{139}$$

where $a^2 = 8m_o$ and m_o is the total variance of a record. Combining equations (138) and (139) and solving for r, one obtains

$$\ln r = -\left(\frac{1}{8.0}\, m_o^{1/2}\right) h^2_{(r)} + \ln (N + 1). \tag{140}$$

Thus, a plot of ln r versus the wave height squared should yield a straight line fitted by the parameters, m_o and N, if the tested wave heights are obtained from a sample which is Rayleigh distributed.

*A typographical error in (138) has been corrected by changing -1 to +1 in the numerator.

"Confidence limits, or probability intervals, can be computed as described by Borgman (1973). These confidence limits can be employed to observe whether the oscillations of the data around the theoretical relationship are causes to accept or reject the theoretical relationship. According to Fisz (1963, pg. 375, equation 10.3.7), the distribution function for the rth largest observation taken from a Rayleigh population would be

$$P_{H(r)}(h) = \sum_{k=0}^{r-1} \binom{N}{k} \left(e^{-h^2/a^2}\right)^{k} \left(1-e^{-h^2/a^2}\right)^{N-k} \tag{141}$$

"To obtain the upper and lower 95% confidence interval, equation 141 is solved for $P_{H(r)}(h)$ equal to 0.975 and 0.025 respectively by the method outlined in the <u>NBS Handbook of Mathematical Functions</u> (1964, pg. 960, examples 18 and 20). The method is briefly outlined below for the upper confidence interval.

Example 18 from the above reference shows

$$\sum_{k=0}^{r-1} \binom{N}{k} p^k q^{N-k} = 1 - \sum_{k=r}^{N} \binom{N}{k} p^k q^{N-k} \tag{142}$$

where $p = e^{-h^2/a^2}$ and $q = 1-p$. The second term on the right hand side of equation (142) is related to the F-distribution as

$$\sum_{k=r}^{N} \binom{N}{k} p^k q^{N-k} = Q(F|\nu_1,\nu_2) \tag{143}$$

where $\nu_1 = 2(N-r+1)$ and $\nu_2 = 2r$. Combining equations (141, 142, 143), and $F_{H(r)}(h) = 0.975$, the following is obtained

$$Q\,(F|\nu_1,\nu_2) = 0.025 \tag{144}$$

"The appropriate F-distribution table is used to determine a value of F based on ν_1 and ν_2, which in turn is used to solve for p by

$$P = \nu_2/(\nu_2 + \nu_1)F \tag{145}$$

The upper confidence point is found by

$$h_{up}^2 = -a^2 \ln p \tag{146}$$

"Thus, by varying the rank, r, appropriate values of h_{up} are found which

determine the upper 95% confidence envelope. A similar approach is used to determine the lower envelope employing example 20 of the above reference.

"Two modifications to Borgman's approach were performed for this study. The first was to rank and test all the waves in a record as opposed to Borgman's use of the top 19 waves only. The second was to specify a slope for the data by the use of $a^2 = 8m_o$, Borgman had determined a least squares fit of the data to a straight line to determine a^2. The author believes that this approach is a true test of the Rayleigh distribution as Borgman's approach tests only the relationship of the data to a height squared fit as a^2, the slope is continually changing with each record tested. -----

"The basic data record unit of 450 digitized points (7.5 minutes of data) was tested as a 7.5 minute block and also combined into a 15 minute block and a 30 minute block and then tested. Sample plots of ln r versus height squared for file 1 data are shown in Figures 35 to 38. For four 7.5 minutes records, in Figures 39 and 40 for two 15 minute records and in Figure 41 for one 30 minute record. (Note: The four 7.5 minute records were combined into the two 15 minute records which were in turn combined into the 30 minute record). In the figures, only the top seventy waves were plotted as crosses, but all wave heights within the record were checked to see if the confidence intervals were exceeded. The slope, as determined by a^2, is shown as a straight line for the first 40 points only, while the upper and lower confidence envelopes are shown as the two outrider lines. ------- As can be observed, the inclusion of all plots would make this report too voluminous and (these figures) will be the only ones presented for the Rayleigh distribution.

"The fit to each record was determined by comparing $h_{(r)}$ to the upper envelope height, $h_{up(r)}$, and the lower envelope height, $h_{low(r)}$. If $h_{(r)}$ was greater than $h_{up(r)}$ or lower than $h_{low(r)}$, the record was considered a bad fit. The total number of heights falling outside the confidence envelopes was determined as a percent of the total number of wave heights in the record.

"Figure 37, shows record 55, file 1 which is the only record shown which passes the above described fitting criteria. Figure 36, shows record 54, file 1 in which data falls outside both the upper and lower confidence levels. The fact that most of the data falls below the center line of the figure should not be surprising as the line was determined theoretically by $8\ m_o$ and

Figure 35

Rayleigh Fit, File 1, Record 53, 7.5 Minute

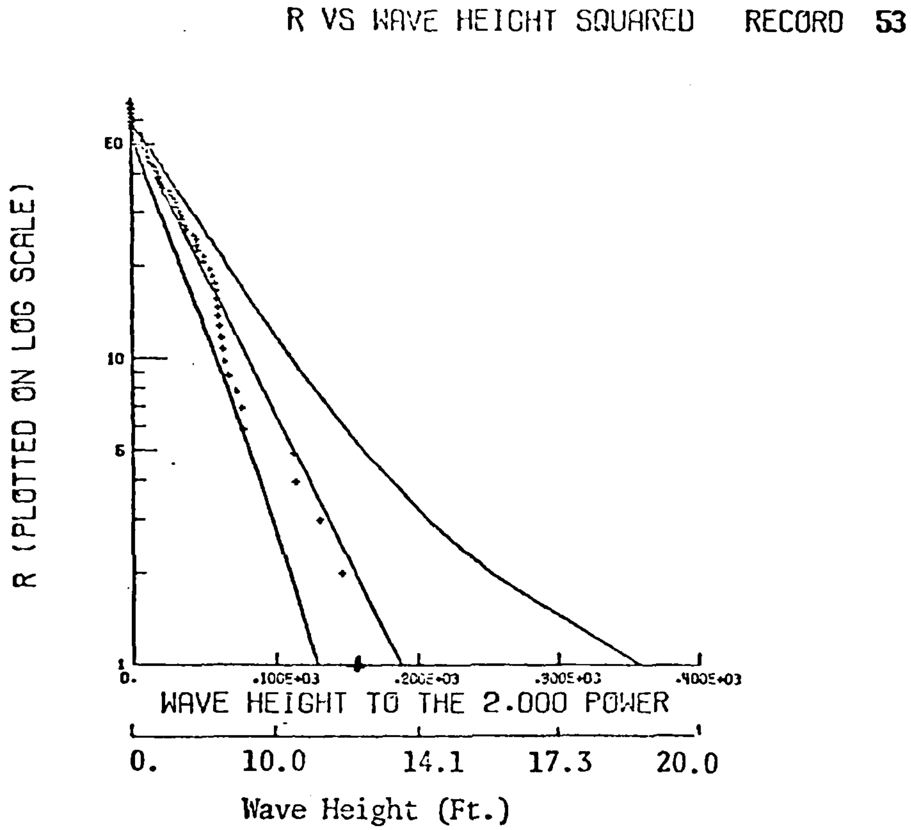


Figure 36

Rayleigh Fit, File 1, Record 54, 7.5 Minute

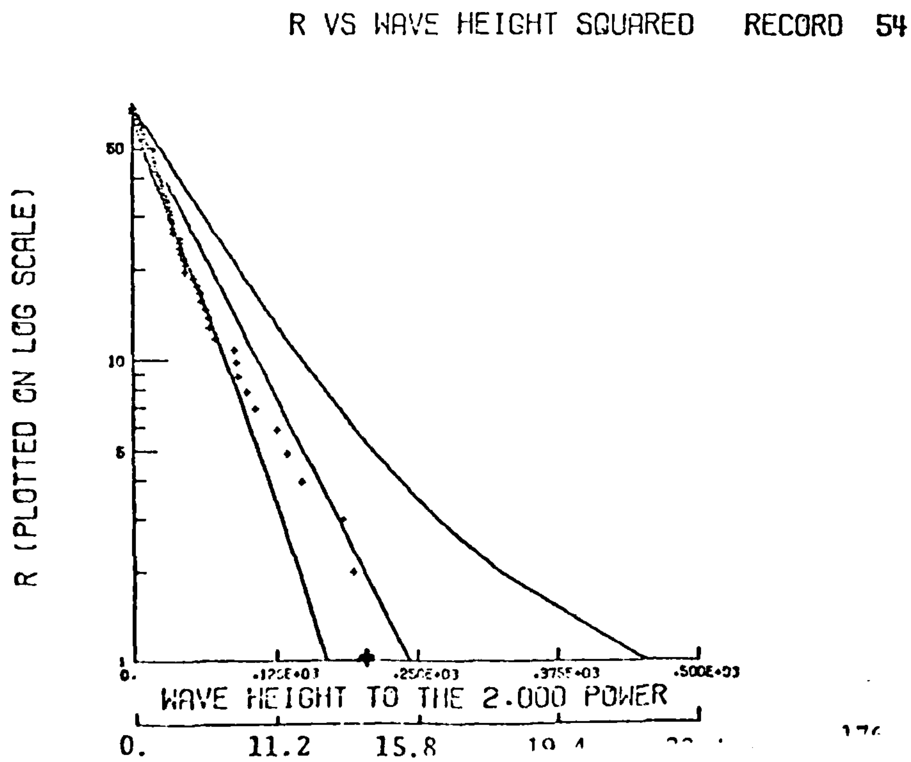


Figure 37

Rayleigh Fit, File 1, Record 55, 7.5 Minute

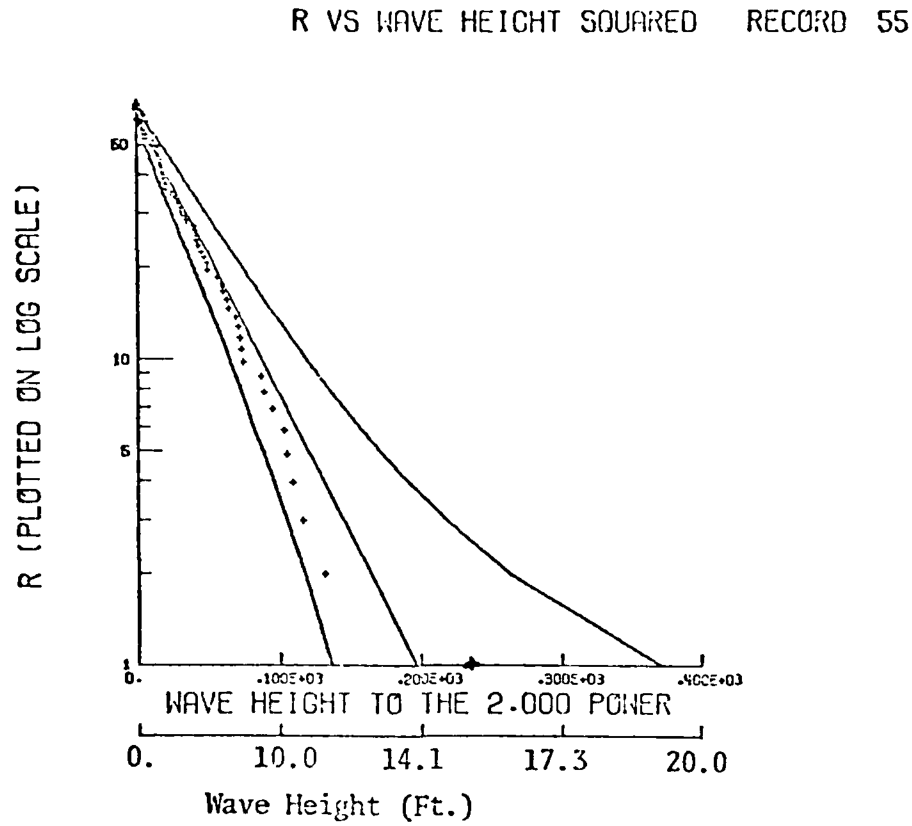


Figure 38

Rayleigh Fit, File 1, Record 56, 7.5 Minute

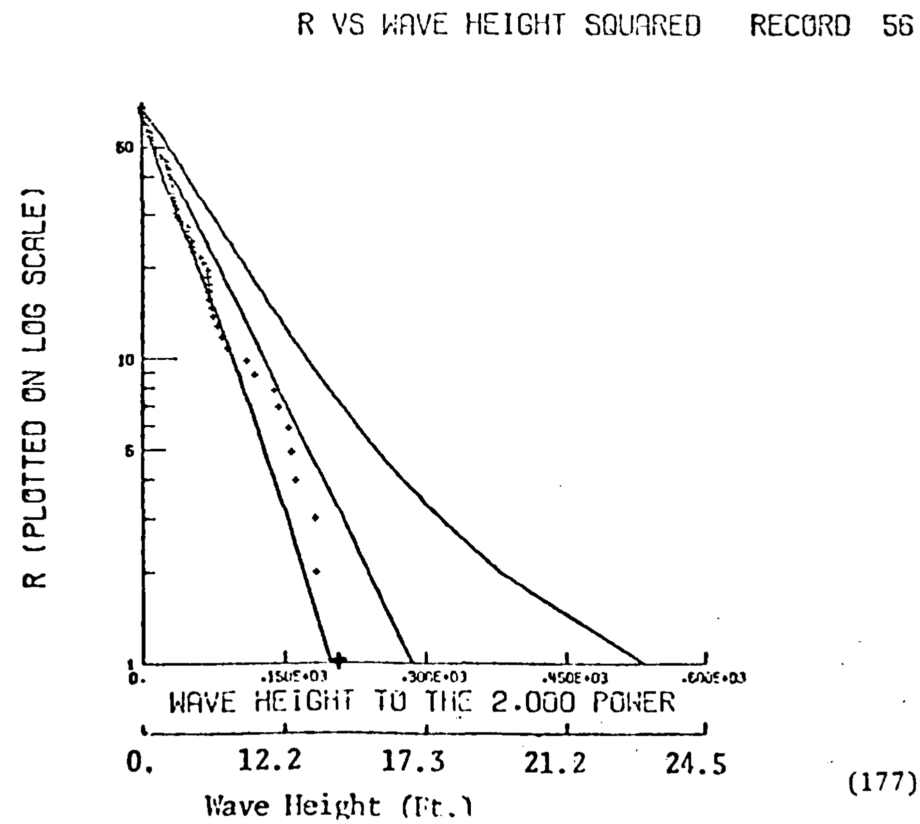


Figure 39

Rayleigh Fit, File 1, Record 27, 15 Minute

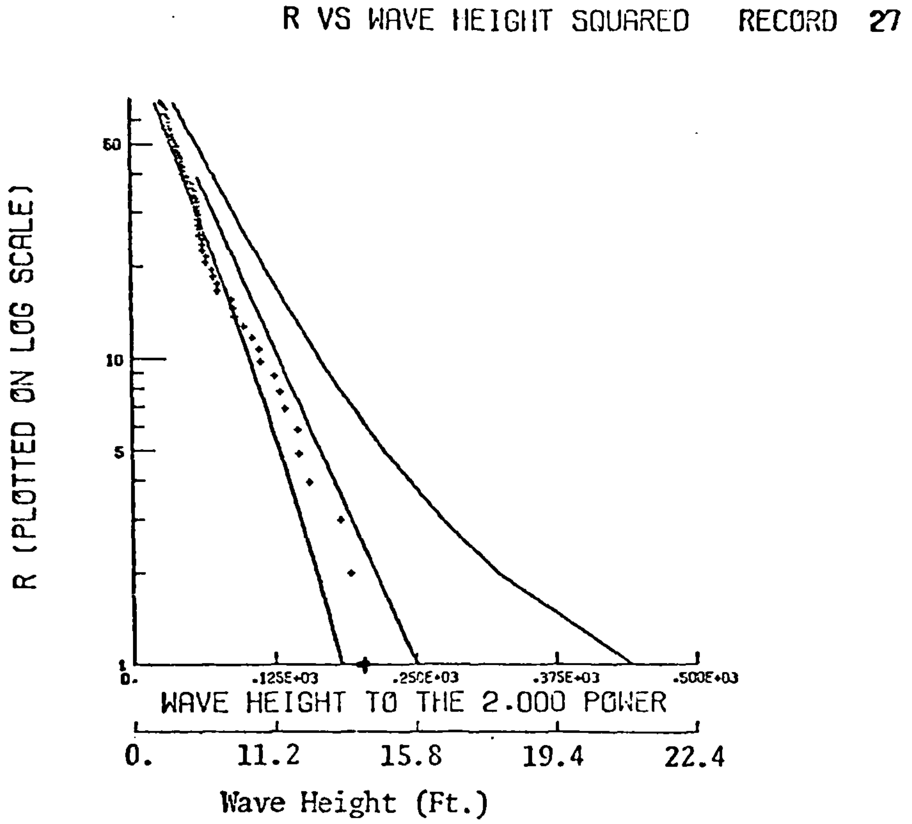
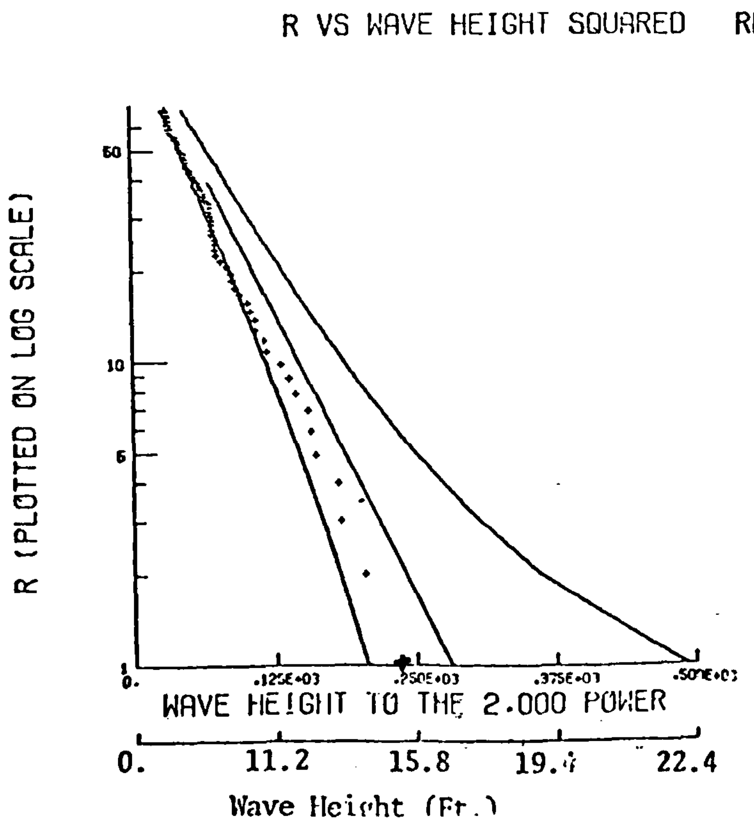


Figure 40

Rayleigh Fit, File 1, Record 28, 15 Minute



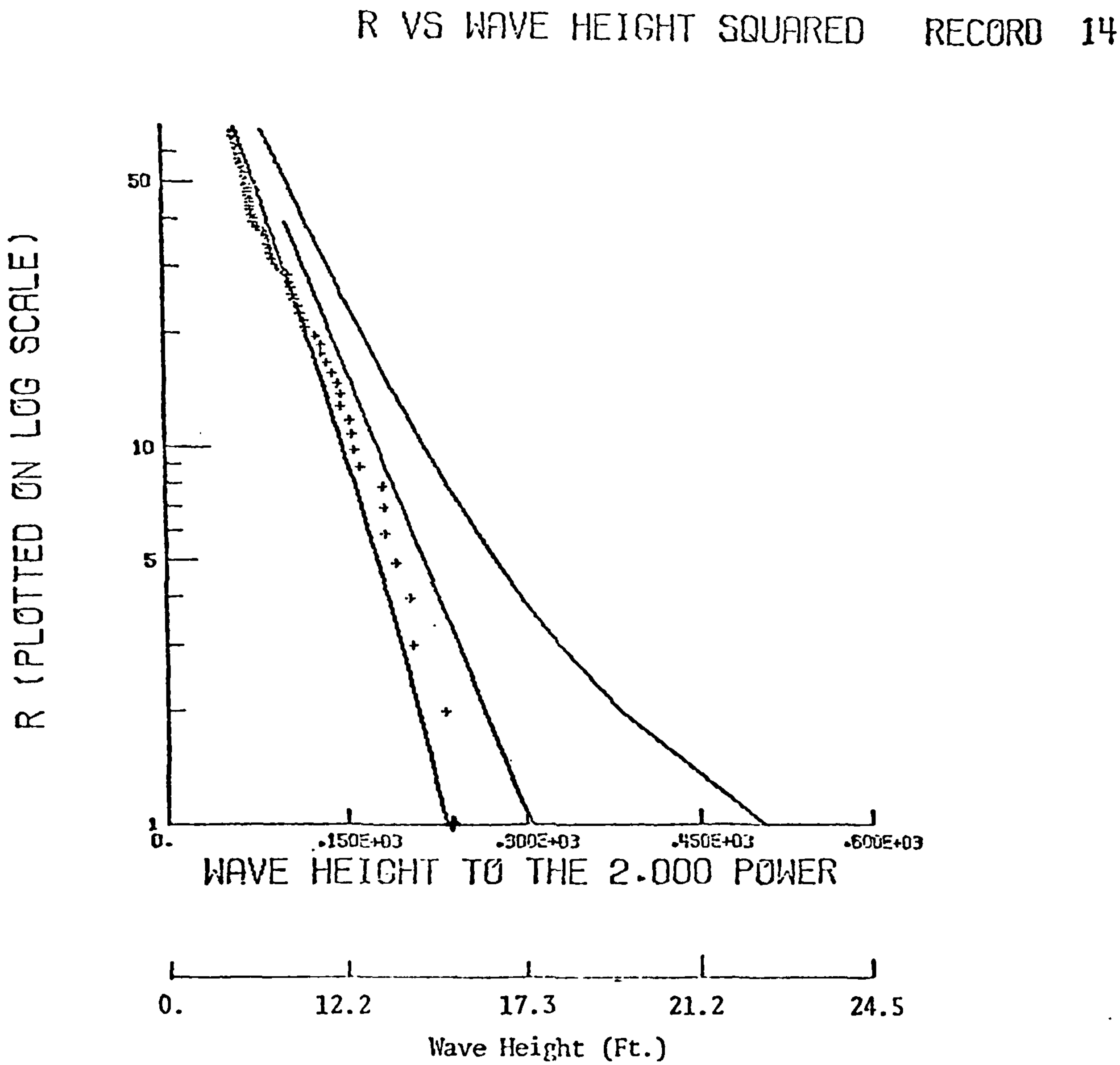


Figure 41
Rayleigh Distribution, File 1, Record 14, 30 Minute

not by a least squares fit to the data. The upper confidence envelope in Figure 36 is exceeded at points 65 through 70, while data falls below the lower lower confidence envelope for points 11 through 30. This type of occurrence was not uncommon and always occurred with the upper limit being exceeded at the low wave height tail of the record and the lower limit being exceeded at the start or middle of the record. This is a mild example of a record exceeding both confidence intervals but there are many more showing a much worse fit. Figure 40 shows the result of combining two 7.5 minute records (numbers 55 and 56), one of which failed the test and the second passing the test, into one 15 minute record (number 28) which fails the test criteria. Figure 39 shows that combining two 7.5 minute records (numbers 53 and 54), which just barely fail the test, into one 15 minute record could result in a fit worse than either of the 7.5 minute fits. Figure 41 shows the four 7.5 minute records, or the two 15 minute records, combined into one 30 minute record which fails the test. In all figures, the first point plotted is right on the X axis and can easily be spotted as being the only tick mark without any scale variable beneath it.

"Table 16 presents the results of this analysis for the 7.5 minute records of File 1 with the following notation:

- H_s: The significant wave height as determined by "bump count".
- SWP: The spectral width parameter as determined by $|T_c$ and $T_z|$.
- #WAVES: The total number of waves of the record to be analyzed.
- RH_s: The significant wave height as determined by 4.005 m_o 1/2.
- Y/N: Fit of the data to the Rayleigh distribution (Y = yes, N = no).
- H/L: Where the data exceeds the 95% confidence envelope (H=upper curve, L=lower curve).
- %: Total percent of points falling outside the 95% confidence envelope.

"Table 17, presents the results for the 15 minute records of File 1 and Table 18 the results for the 30 minute records.

"Summary results of Tables 16, 17 and 18 are presented in Tables 19, 20 and 21. ----- The tables show the number of records passing or failing by five foot increments of significant wave height. The heading all points shows the number of records that had at least one point fall outside the 95% confidence envelope and the heading greater than 5% shows the number of

records that had at least 5% of the points fall outside the confidence envelope. The numbers under H and L show how many records exceeded the upper and lower envelope. If a record exceeded both the upper and lower confidence envelope (as shown in Figure 17), it was counted under both H and L, thus the combined value of H and L could exceed the number of records failed. -----"

The material that has just been quoted represents a serious attempt to find out whether or not crest to trough wave heights fit the Rayleigh pdf. It also found the parameters appropriate to equation (114), but these will not be considered. Many of the other published examples simply compare the sample distribution with the theoretical distribution, or with a family of distributions, and the lack of high waves is considered to be prima facie evidence that the Raleigh pdf is not a good fit and that whatever new model is proposed is superior.

At the time of the study by Spring (1978), which was a doctoral dissertation, his faculty advisor (W. J. Pierson), was unaware of the implications of the various assumptions inherent in the material quoted from this work. The definition of the sample cdf differs from the accepted one given in Mood, Grabill, and Boes (1963). The equation for the distribution of the sample cdf is correctly stated, but the approximation by the F distribution is probably valid only for large samples. A confidence interval from 0.025 to 0.975 may not be correctly calculated for the smaller samples. The value of m_o (E_o) calculated from the spectrum may be too high because of digitization noise. The samples being tested are not random samples, and the effective number of independent wave heights is less than the value of N that was used.

A somewhat smaller value of m_o, to account for digitization noise may be needed. Wider confidence intervals are needed to account for a smaller effective sample. They should be actually based on the binomial distribution, and not an approximation to it. These changes would confine more of the sample cdf's within the wider acceptance interval.

With these points in mind, consider the results for the 7.5 minute samples, the 15 minute samples and the 30 minute samples. For the 180, 7.5 minute samples, the first 35 resulted in the rejection of the hypothesis

TABLE 16a

FILE 1, 7.5 MINUTE ANALYSIS

REC	H_S	SWP	#WAVES	RH_S	Y/N	H/L	%	FH_S	Y/N	H/L	%
1	3.29	.224308	96	3.67	N	L	26.0	3.45	N	L	2.1
2	3.23	.315101	96	3.70	N	L	11.5	3.49	N	L	1.0
3	3.19	.305556	90	3.60	N	L	1.1	3.39	Y		
4	3.12	.265306	96	3.72	N	L	19.8	3.51	N	L	14.6
5	3.10	.305556	90	3.62	N	L	18.9	3.41	N	L	1.1
6	3.22	.374463	87	3.22	N	L	5.8	3.46	N	L	1.2
7	3.02	.195039	93	3.71	N	L	31.2	3.50	N	L	20.4
8	3.51	.305556	90	3.90	N	L	5.6	3.67	Y		
9	3.77	.372019	84	4.21	N	H/L	2.4/9.5	3.97	N	H	1.2
10	3.71	.318239	90	4.27	N	L	7.8	4.02	N	L	3.3
11	3.81	.305556	90	4.36	N	L	11.1	4.11	N	L	4.4
12	3.96	.330579	90	4.33	N	L	4.4	4.08	N	L	3.3
13	3.98	.338894	87	4.66	N	L	16.1	4.39	N	L	4.6
14	3.76	.362932	87	4.30	N	L	6.9	4.05	N	L	2.3
15	3.78	.286549	87	4.60	N	L	26.4	4.34	N	L	10.3
16	3.72	.362932	87	4.52	N	L	21.8	4.26	N	L	16.1
17	4.25	.383702	84	4.73	N	L	2.4	4.46	Y		
18	4.42	.393399	81	4.96	N	L	11.1	4.68	Y		
19	4.50	.308303	84	5.22	N	L	14.3	4.92	N	L	2.4
20	4.37	.415225	78	4.83	N	L	2.6	4.55	N	L	2.6
21	4.37	.321799	84	4.97	N	L	10.7	4.69	Y		
22	4.66	.437500	78	5.37	N	L	21.8	5.06	Y		
23	4.60	.334904	84	5.18	N	L	19.1	4.88	N	L	6.0
24	5.05	.426079	75	5.59	N	L	12.0	5.27	Y		
25	6.23	.504269	69	7.00	N	L	4.4	6.59	Y		
26	5.55	.426079	75	6.54	N	L	10.7	6.17	N	L	4.0
27	6.22	.449532	69	7.24	N	L	8.7	6.82	N	L	1.5
28	5.47	.426079	75	6.45	N	L	36.0	6.07	N	L	10.7
29	5.71	.402168	75	6.79	N	L	13.3	6.40	N	L	8.0
30	5.81	.576273	69	6.52	N	L	1.5	6.14	Y		
31	5.85	.403588	78	6.85	N	L	28.2	6.46	N	L	16.7
32	6.71	.493995	69	7.46	N	L	1.5	7.03	Y		
33	6.69	.335421	75	7.47	N	L	8.0	7.04	N	L	1.3
34	6.48	.993995	69	7.02	N	L	4.4	6.62	N	L	4.4
35	6.91	.483398	69	7.76	N	L	5.8	7.31	N	L	1.5

TABLE 16b

FILE 1, 7.5 MINUTE ANALYSIS
Page 2

REC	H_S	SWP	#WAVES	RH_S	Y/N	H/L	%	FH_S	Y/N	H/L	%
36	7.18	.483398	69	8.72	N	L	13.0	7.75	N	L	2.9
37	6.78	.425596	72	8.08	N	L	23.6	7.61	N	L	16.7
38	6.77	.305556	75	7.87	N	L	18.7	7.42	Y		
39	7.29	.550815	63	8.15	N	L	6.4	7.68	N	L	1.6
40	6.57	.414307	75	7.55	N	L	28.0	7.11	N	L	9.3
41	7.22	.389648	75	7.99	N	L	12.0	7.53	N	L	16.0
42	6.70	.402168	75	7.54	N	L	5.3	7.11	Y		
43	7.99	.565270	60	9.23	N	L	3.3	8.70	Y		
44	8.10	.483398	69	8.94	N	L	18.8	8.43	N	L	8.7
45	7.13	.472465	69	7.95	N	L	1.5	7.49	Y		
46	7.86	.449038	72	8.63	N	L	27.2	8.13	N	L	27.2
47	9.23	.463359	63	10.34	N	L	14.3	9.75	N	L	12.7
48	7.74	.248889	78	8.87	N	L	65.4	8.36	N	L	53.9
49	7.82	.426079	75	9.01	N	L	10.7	8.49	N	L	2.7
50	9.39	.566330	54	10.28	N	H/L	5.6/1.9	9.69	N	H	5.6
51	8.15	.531073	63	9.22	N	L	3.2	8.69	Y		
52	9.26	.437500	63	9.96	N	L	4.8	9.39	N	H	4.8
53	8.89	.570749	57	9.65	N	H	3.5	9.10	N	H	1.8
54	9.50	.473977	66	10.76	N	H/L	1.5/18.2	10.14	N	L	3.0
55	9.15	.531073	63	9.75	Y			9.19	Y		
56	10.30	.411033	66	11.67	N	L	12.1	10.99	N	L	1.5
57	9.31	.564408	66	10.03	Y			9.45	Y		
58	9.71	.370987	69	10.84	N	L	24.6	10.21	N	L	20.3
59	11.19	.528379	57	11.74	Y			11.07	Y		
60	9.61	.349636	75	10.54	N	L	40.0	9.93	N	L	37.3
61	10.31	.382653	66	11.30	N	L	7.6	10.65	N	L	1.5
62	10.26	.987474	63	11.09	Y			10.45	Y		
63	9.30	.496358	66	10.28	Y			9.68	Y		
64	9.16	.541103	63	10.81	N	L	11.1	10.19	N	L	4.8
65	10.17	.423864	63	11.17	Y			10.52	Y		
66	8.62	.504269	69	10.16	N	L	15.5	9.57	N	L	4.4
67	9.09	.363400	75	10.71	N	L	48.0	10.09	N	L	28.0
68	9.42	.376731	75	10.70	N	L	4.0	10.08	N	L	4.0
69	10.06	.385201	69	11.31	N	L	2.9	10.65	Y		
70	9.93	.924495	66	10.91	N	L	1.5	10.29	N	L	1.5

TABLE 16c

FILE 1, 7.5 MINUTE ANALYSIS
Page 3

REC	H_S	SWP	#WAVES	RH_S	Y/N	H/L	%	FH_S	Y/N	H/L	%
71	10.44	.545512	60	11.08	Y			10.44	N	H	1.7
72	10.62	.425069	69	11.60	N	L	1.5	10.93	N	L	1.5
73	9.80	.424495	66	11.25	N	H/L	3.0/9.1	10.60	N	H/L	3.0/4.6
74	11.34	.367689	66	12.35	Y			11.64	Y		
75	11.23	.437500	63	12.47	N	L	1.6	11.75	Y		
76	10.01	.425596	72	11.02	N	L	26.4	10.38	N	L	11.1
77	10.98	.411033	66	12.26	N	L	6.1	11.55	N	L	3.0
78	11.46	.450657	63	12.45	N	H/L	11.1/1.6	11.74	N	H	7.9
79	11.49	.423169	60	12.48	Y			11.76	Y		
80	10.91	.501730	60	11.85	Y			11.16	Y		
81	10.90	.409726	63	12.11	N	L	1.6	11.41	Y		
82	9.74	.397093	66	11.47	N	L	15.2	10.80	Y		
83	10.89	.274348	69	12.40	N	L	2.9	11.68	Y		
84	11.74	.510000	63	12.99	Y			12.24	Y		
85	11.95	.524376	60	12.94	N	L	3.3	12.20	N	L	1.7
86	12.05	.570749	57	13.03	Y			12.27	N	L	1.8
87	10.92	.464604	60	12.07	N	H	1.7	11.38	N	H	1.7
88	12.28	.555556	60	13.53	N	H/L	1.7/1.7	12.75	Y		
89	11.84	.463359	63	12.89	Y			12.15	Y		
90	11.66	.487474	63	13.50	N	L	19.1	12.73	N	H	1.6
91	12.38	.376731	60	13.20	Y			12.44	Y		
92	12.81	.423169	60	14.05	N	L	3.3	13.24	Y		
93	12.01	.450657	63	13.38	Y			12.61	Y		
94	11.63	.535124	60	12.46	Y			11.74	Y		
95	10.48	.274348	69	11.99	N	L	40.6	11.30	N	L	27.5
96	12.53	.450657	63	13.95	N	L	7.9	13.14	N	H/L	1.6/3.2
97	13.06	.539541	57	14.94	N	L	1.8	14.08	Y		
98	11.50	.437500	63	12.37	N	L	1.6	11.65	N	L	1.6
99	11.99	.539541	57	13.58	N	L	1.8	12.79	N	L	1.8
100	12.93	.528379	57	14.90	N	L	3.5	14.04	Y		
101	13.91	.464404	60	14.41	N	L	3.3	14.61	N	L	1.7
102	12.90	.379844	63	14.04	Y			13.[illegible]3	Y		
103	14.34	.364044	63	14.45	N	L	4.8	14.56	N	L	11.1
104	14.80	.495152	54	15.76	Y			14.85	Y		
105	14.05	.463359	63	14.59	N	L	30.2	13.75	N	H/L	1.6/19.1

TABLE 16d

FILE 1, 7.5 MINUTE ANALYSIS

Page 4

REC	H_S	SWP	#WAVES	RH_S	Y/N	H/L	%	FH_S	Y/N	H/L	%
106	14.56	.475624	63	16.26	N	L	34.0	15.33	N	L	25.4
107	13.47	.504801	57	15.70	N	L	14.0	14.79	N	L	3.5
108	14.16	.437500	60	15.81	Y			14.89	Y		
109	14.12	.463359	60	15.32	Y			14.43	Y		
110	15.79	.373264	57	17.43	Y			16.43	Y		
111	14.20	.492344	57	16.83	N	L	15.8	15.86	N	L	7.0
112	13.70	.390317	57	15.31	N	L	1.8	14.43	Y		
113	16.25	.465976	57	17.54	N	L	19.3	16.53	N	L	19.3
114	15.07	.504801	57	16.56	Y			15.60	Y		
115	14.47	.422400	57	16.60	N	L	8.8	15.64	N	L	8.8
116	17.79	.511916	51	18.68	Y			17.60	Y		
117	15.53	.525018	51	16.87	N	H/L	15.7/2.0	15.89	N	H	19.6
118	15.56	.5081809	54	17.43	N	L	1.9	16.42	Y		
119	17.91	.511916	51	19.14	N	L	35.3	18.04	N	L	23.5
120	15.68	.421543	54	17.14	Y			16.15	Y		
121	21.10	.419501	48	21.89	Y			20.63	Y		
122	17.24	.453686	51	18.90	N	H/L	2.0/3.9	17.81	Y		
123	18.13	.369377	54	19.43	Y			18.31	Y		
124	19.00	.525018	51	20.84	Y			19.64	Y		
125	16.81	.297362	57	18.57	Y			17.50	Y		
126	19.63	.549688	51	20.75	Y			19.56	Y		
127	20.15	.525018	51	21.63	Y			20.38	Y		
128	22.90	.555556	48	24.81	Y			23.38	Y		
129	20.42	.437500	51	23.54	N	L	5.9	22.19	Y		
130	21.56	.537600	51	22.56	Y			21.62	Y		
131	22.45	.453686	51	24.25	Y			22.85	Y		
132	20.44	.453686	41	22.77	Y			21.46	Y		
133	21.81	.542948	48	22.69	Y			21.38	Y		
134	22.77	.529796	48	25.00	N	L	2.1	23.56	N	H/L	2.1/2.1
135	20.91	.574669	45	23.45	Y			22.10	Y		
136	27.17	.562067	45	27.92	N	L	35.6	26.31	N	L	33.3
137	26.00	.520710	45	27.44	Y			25.86	Y		
138	29.43	.542948	48	25.82	Y			24.39	Y		
139	28.24	.520710	45	30.84	N	L	2.2	29.06	N	L	2.2
140	24.01	.555556	48	26.91	Y			25.36	Y		

TABLE 16e

FILE 1, 7.5 MINUTE ANALYSIS
Page 5

REC	H_S	SWP	#WAVES	RH_S	Y/N	H/L	%	FH_S	Y/N	H/L	%
141	28.99	.525934	42	30.57	Y			28.80	Y		
142	29.41	.419501	48	32.11	N	L	27.1	30.25	N	L	27.1
143	25.91	.574669	45	28.84	N	H	2.2	27.18	N	H	2.2
144	29.94	.618512	42	32.19	Y			30.33	Y		
145	39.07	.493249	42	39.81	N	L	21.4	37.52	N	H/L	2.4/23.8
146	26.93	.562067	45	28.33	N	L	2.2	26.70	Y		
147	24.98	.473205	45	28.3	N	H/L	15.6/2.2	26.69	N	H	20.0
148	29.11	.535124	45	32.05	N	L	2.2	30.20	N	L	2.2
149	33.06	.455791	45	35.77	N	L	2.2	33.71	N	L	4.4
150	25.47	.469184	51	28.73	N	L	35.3	27.08	N	L	27.5
151	34.12	.547860	39	34.84	Y			32.83	N	L	2.6
152	29.94	.338121	48	32.01	N	L	6.3	30.16	N	L	2.1
153	28.32	.520710	45	30.72	N	L	2.2	28.95	Y		
154	33.85	.221453	45	36.16	Y			34.07	N	H/L	2.2/2.2
155	32.26	.541103	42	33.18	N	L	9.5	31.27	N	L	9.5
156	33.60	.591239	39	34.93	N	H	2.6	32.92	N	H	7.7
157	40.21	.664820	33	40.88	Y			38.52	Y		
158	39.16	.618512	42	38.84	Y			36.60	N	H	2.4
159	29.25	.607039	42	31.32	Y			29.43	Y		
160	31.26	.555556	42	35.31	N	H	4.8	33.28	N	H	4.8
161	35.06	.486745	48	37.32	N	L	25.0	35.17	N	L	27.1
162	39.18	.562067	45	34.90	Y			32.88	Y		
163	29.13	.582485	42	32.06	N	H	26.2	30.21	N	H	31.0
164	32.99	.505615	45	36.27	Y			34.18	Y		
165	31.21	.535124	45	33.03	N	L	2.2	31.12	N	L	6.7
166	35.81	.380812	48	38.33	N	L	12.5	36.12	N	L	16.7
167	34.05	.505615	45	35.95	Y			33.88	Y		
168	36.05	.437500	45	37.68	Y			35.51	Y		
169	37.57	.473205	45	39.57	N	L	2.2	37.38	Y		
170	39.42	.541103	42	39.55	N	H	2.5	37.27	N	H	2.4
171	32.35	.525934	42	34.94	Y			32.92	N	H	4.8
172	45.10	.478395	39	44.61	Y			42.04	N	H	5.1
173	38.69	.398038	45	40.11	N	L	2.2	37.80	N	L	2.2
174	38.37	.437500	45	40.01	Y			37.71	Y		
175	43.07	.555556	42	44.20	Y			41.65	Y		

TABLE 16f

FILE 1, 7.5 MINUTE ANALYSIS
Page 6

REC	H_S	SWP	#WAVES	RH_S	Y/N	H/L	%	FH_S	Y/N	H/L	%
176	47.38	.437500	45	47.94	N	L	11.1	45.17	N	L	11.1
177	41.37	.505615	45	43.63	Y			41.12	Y		
178	42.43	.473205	45	42.99	Y			40.51	Y		
179	33.23	.384379	51	35.41	N	L	2.0	33.37	N	L	2.0
180	50.24	.478395	39	52.29	N	H	5.1	49.27	N	L	7.7

TABLE 17a

FILE 1, 15 MINUTE ANALYSIS

REC	H_S	SWP	#WAVES	RH_S	Y/N	H/L	%	FH_S	Y/N	H/L	%
1	3.26	.255521	195	3.69	N	H/L	15/61.0	3.48	N	L	20.5
2	3.16	.245224	192	3.66	N	L	24.3	3.45	N	L	12.7
3	3.18	.340775	177	3.65	N	L	26.6	3.44	N	L	6.8
4	3.30	.227236	189	3.80	N	L	21.5	3.58	N	L	9.7
5	3.74	.322250	177	4.24	N	L	15.3	4.00	N	L	6.8
6	3.88	.295324	183	4.34	N	L	19.4	4.09	N	L	2.2
7	3.88	.328511	177	4.48	N	L	23.6	4.23	N	L	6.3
8	3.78	.302933	177	4.56	N	H/L	1.2/32.8	4.30	N	H/L	1.2/20.1
9	4.35	.366052	168	4.85	N	L	21.2	4.57	N	L	.6
10	4.46	.339343	165	5.03	N	L	19.1	4.74	N	L	1.2
11	4.52	.358446	165	5.17	N	L	33.3	4.87	N	L	5.6
12	4.78	.339343	165	5.39	N	L	22.0	5.08	N	L	2.5
13	5.94	.465691	144	6.77	N	L	17.4	6.38	N	L	5.6
14	5.88	.419877	147	6.85	N	L	28.5	6.46	N	L	4.9
15	5.82	.496809	144	6.66	N	L	21.8	6.27	N	L	6.1
16	6.31	.454332	147	7.16	N	L	35.4	6.75	N	L	6.8
17	6.60	.401413	147	7.25	N	L	19.7	6.83	N	L	5.4
18	7.13	.488738	138	7.99	N	L	13.8	7.53	N	L	5.1
19	6.77	.342586	150	7.97	N	L	46.9	7.51	N	L	24.5
20	6.98	.466268	141	7.85	N	L	33.3	7.40	N	L	10.6
21	6.92	.395957	150	7.77	N	L	16.0	7.32	N	L	6.0
22	8.15	.507017	132	9.09	N	L	17.8	8.56	N	L	1.6
23	7.44	.460693	141	8.30	N	L	26.2	7.82	N	L	10.6
24	8.53	.358181	141	9.63	N	L	82.3	9.08	N	L	64.5
25	8.67	.473977	132	9.67	N	L	11.4	9.11	N	L	.8
26	8.72	.468831	120	9.60	N	L	.8	9.04	N	L	.8
27	9.11	.504510	126	10.22	N	H/L	6.5/10.6	9.63	N	H/L	5.7/1.6
28	9.70	.450069	132	10.75	N	L	9.3	10.13	Y		
29	9.56	.484354	135	10.44	N	L	32.6	9.84	N	L	19.3
30	10.41	.437500	132	11.16	N	L	11.4	10.51	N	L	6.1
31	10.29	.437500	129	11.19	N	L	3.9	10.55	N	L	3.1
32	9.21	.496358	132	10.55	N	L	17.1	9.94	N	L	3.1
33	9.47	.443698	135	10.67	N	L	13.6	10.06	Y		
34	9.40	.376731	150	10.70	N	L	37.3	10.09	N	L	9.3
35	10.02	.358201	133	11.11	N	L	17.8	10.47	Y		

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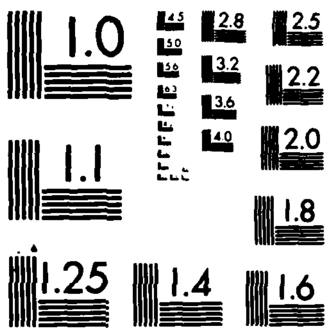
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TABLE 17b

FILE I, 15 MINUTE ANALYSIS
Page 2

REC	H_S	SWP	#WAVES	RH_S	Y/N	H/L	%	FH_S	Y/N	H/L	%
36	10.48	.462222	132	11.34	N	L	3.9	10.69	Y		
37	10.53	.397093	132	11.81	N	H/L	3.0/11.4	11.13	N	L	1.5
38	10.79	.437500	135	11.77	N	L	4.4	11.09	N	L	.7
39	11.22	.437500	129	12.36	N	L	7.8	11.65	N	L	2.3
40	11.29	.444298	123	12.17	N	H/L	.8/2.4	11.47	N	H	.8
41	10.44	.382653	132	11.79	N	L	11.4	11.11	N	L	.8
42	11.42	.411033	132	12.70	N	L	13.6	11.97	N	L	.8
43	11.93	.529796	120	12.99	N	L	4.2	12.24	N	H/L	3.3/.8
44	11.65	.518861	120	12.82	N	H/L	4.2/7.5	12.08	N	L	4.8
45	11.78	.475624	126	13.20	N	L	15.1	12.44	Y		
46	12.53	.378328	123	13.63	N	L	7.5	12.84	Y		
47	11.74	.469545	126	12.93	N	L	1.6	12.18	Y		
48	11.58	.338620	135	13.00	N	L	56.1	12.25	N	H/L	.8/28.0
49	12.60	.470292	123	13.71	N	L	4.1	12.92	N	L	.8
50	12.57	.509161	117	14.26	N	L	19.7	13.43	N	L	.9
51	13.41	.423525	123	14.79	N	L	4.1	13.94	Y		
52	14.80	.408284	120	15.61	N	L	2.6	14.71	N	L	3.4
53	14.27	.443984	129	15.45	N	L	60.3	14.56	N	H/L	.8/60.3
54	14.07	.444466	120	15.75	N	L	12.5	14.84	Y		
55	14.81	.423169	120	16.41	N	L	3.4	15.46	Y		
56	14.37	.384852	120	16.09	N	L	22.2	15.16	Y		
57	15.64	.492344	114	17.06	N	L	29.8	16.07	N	L	28.1
58	16.49	.437500	111	17.67	N	L	5.4	16.65	N	L	.9
59	15.57	.516469	105	17.15	N	H/L	1.9/2.9	16.16	N	H/L	1.9/1.0
60	16.77	.445232	108	18.17	N	L	4.8	17.12	N	L	3.8
61	19.09	.402893	102	20.45	N	L	1.0	19.27	Y		
62	18.62	.421543	108	20.15	N	L	.9	18.98	Y		
63	18.38	.405816	111	19.69	N	L	5.6	18.56	N	L	3.7
64	21.58	.450205	99	23.27	Y			21.93	Y		
65	20.90	.460854	105	23.06	N	L	6.7	21.73	N	L	1.0
66	21:37	.421078	105	23.52	N	L	1.0	22.16	Y		
67	22.17	.513936	99	23.87	Y			22.49	Y		
68	24.51	.539187	93	25.79	Y			24.30	N	L	4.3
69	25.10	.501730	96	26.65	Y			25.11	Y		
70	26.59	.477809	99	28.94	Y			27.27	Y		

TABLE 17c

FILE 1, 15 MINUTE ANALYSIS
Page 3

REC	H_S	SWP	#WAVES	RH_S	Y/N	H/L	%	FH_S	Y/N	H/L	%
71	29.57	.437500	93	31.35	N	L	2.2	29.54	N	L	1.0
72	27.96	.568437	90	30.57	N	H/L	1.2/1.2	28.80	Y		
73	33.07	.497799	90	34.55	N	H/L	1.2/35.6	32.56	N	H/L	1.2/36.8
74	26.91	.505615	90	30.24	N	H/L	2.1/1.1	28.5	N	H	2.2
75	29.67	.462968	96	32.45	N	L	27.1	30.58	N	L	26.0
76	31.89	.447074	87	33.45	Y			31.52	N	L	1.2
77	31.11	.398038	90	33.55	N	L	2.2	31.61	N	H/L	1.1/2.2
78	32.65	.566330	81	34.07	N	L	2.5	32.11	N	H/L	6.2/6.2
79	40.62	.610624	78	39.87	Y			37.57	N	H/L	1.3/1.3
80	30.86	.552130	87	33.34	Y			31.41	N	H	3.5
81	34.41	.494321	96	36.13	N	L	27.1	34.05	N	L	29.2
82	30.99	.513250	90	34.23	N	H/L	13.3/2.2	32.25	N	H	18.9
83	33.52	.463761	93	35.78	N	L	6.5	33.72	N	L	8.6
84	35.29	.437500	93	36.83	Y			34.70	Y		
85	38.26	.473205	90	39.56	N	H/L	1.1/1.1	37.28	N	H	1.1
86	38.70	.427675	87	40.07	Y			37.76	Y		
87	38.87	.418271	90	40.06	Y			37.75	Y		
88	44.94	.464604	90	46.11	N	L	11.1	43.45	N	L	17.8
89	41.56	.455215	93	43.32	Y			40.82	Y		
90	42.68	.437500	90	44.66	Y			42.09	N	H/L	1.1/1.1

TABLE 18a

FILE 1, 30 MINUTE ANALYSIS

REC	H_S	SWP	#WAVES	RH_S	Y/N	H/L	%	FH_S	Y/N	H/L	%
1	3.19	.230464	393	3.69	N	H/L	3.1/62.2	3.48	N	H/L	1.6/39.6
2	3.24	.273765	369	3.73	N	L	35.3	3.51	N	L	19.6
3	3.79	.285526	366	4.30	N	H/L	.3/39.8	4.05	N	L	10.1
4	3.83	.319087	354	4.52	N	H/L	.3/38.3	4.26	N	L	24.8
5	4.38	.356139	333	4.94	N	L	33.3	4.65	N	L	5.8
6	4.68	.349000	330	5.28	N	L	50.6	4.98	N	L	12.0
7	5.91	.446099	291	6.81	N	L	34.7	6.42	N	L	14.8
8	6.02	.465137	294	6.91	N	L	41.5	6.52	N	L	22.8
9	6.85	.425720	291	7.63	N	L	40.4	7.19	N	L	7.0
10	6.88	.398250	294	7.19	N	L	59.8	7.46	N	L	40.6
11	7.53	.440449	285	8.45	N	L	49.5	7.97	N	L	24.0
12	8.04	.400217	285	8.99	N	L	79.8	8.47	N	L	66.0
13	8.69	.462222	264	9.63	N	L	18.0	9.08	N	L	.4
14	9.42	.480634	258	10.99	Y			9.88	N	H	3.1
15	10.03	.452805	270	10.80	N	L	44.1	10.18	N	L	23.7
16	9.82	.459222	264	10.88	N	L	45.4	10.25	N	L	28.6
17	9.43	.384769	291	10.69	N	L	7.3	10.07	Y		
18	10.25	.428006	270	11.22	N	H/L	11.4/.4	10.58	N	H	16.3
19	10.66	.404898	270	11.79	N	L	7.6	11.11	N	L	1.1
20	11.25	.430824	255	12.26	N	L	53.0	11.56	N	L	15.5
21	11.06	.397093	264	12.25	N	L	4.1	11.55	N	H	5.4
22	11.86	.515201	243	12.90	N	L	.8	12.16	Y		
23	12.32	.416860	252	13.42	N	L	69.8	12.64	N	L	40.8
24	11.65	.393531	264	12.97	N	L	2.4	12.22	N	H	.4
25	12.57	.476961	243	13.99	N	L	.4	13.18	N	L	.4
26	14.11	.401566	246	15.21	N	L	15.4	14.33	N	L	.8
27	14.27	.4442.6	249	15.60	N	L	69.9	14.70	N	L	50.4
28	14.68	.404473	240	16.25	N	L	34.6	15.31	Y		
29	16.07	.437500	231	17.37	N	L	29.4	16.37	N	L	29.4
30	16.22	.485665	213	17.67	N	L	6.2	16.65	N	L	1.9
31	18.83	.395692	213	20.30	N	L	4.8	19.13	Y		
32	20.01	.464236	213	21.56	N	L	22.4	20.31	N	L	18.6
33	21.14	.441496	210	23.29	N	L	17.4	21.95	Y		
34	23.34	.526419	192	24.85	Y			23.41	N	L	3.7
35	25.96	.493513	195	27.82	N	L	6.3	26.21	Y		

TABLE 18b

FILE 1, 30 MINUTE ANALYSIS
Page 2

REC	H_S	SWP	#WAVES	RH_S	Y/N	H/L	%	FH_S	Y/N	H/L	%
36	29.08	.508389	183	30.96	N	L	4.4	29.18	Y		
37	30.11	.484983	183	32.47	N	H/L	.6/6.8	30.60	N	H	.6
38	31.08	.442009	186	32.95	N	L	45.4	31.05	N	L	33.9
39	31.95	.474375	174	33.81	N	L	5.9	31.86	N	H/L	.6/2.9
40	35.53	.584579	165	36.75	Y			34.63	Y		
41	32.75	.487474	189	35.19	N	L	5.9	33.16	N	L	5.9
42	34.66	.433009	189	36.31	N	L	.5	34.21	N	L	.5
43	38.45	.451533	177	39.81	N	H	.6	37.52	N	H	.6
44	41.95	.442158	180	43.19	N	L	3.3	40.70	N	H/L	.6/3.9
45	42.29	.451081	183	44.00	Y			41.46	Y		

TABLE 19

FILE I

7.5 MINUTE ANALYSIS SUMMARY

	RAYLEIGH DISTRIBUTION						FORRISTALL DISTRIBUTION					
SIG HEIGHT	# REC.	# REC.	ALL POINTS		G.T. 5%		# REC.	# REC.	ALL POINTS		G.T. 5%	
RANGE	PASSED	FAILED	H	L	H	L	PASSED	FAILED	H	L	H	L
0 - 4.99	0	23	1	23	0	19	6	17	1	16	0	5
5.0- 9.99	2	40	4	39	1	18	12	30	4	27	1	13
10.0-14.99	18	29	3	18	1	12	25	22	6	18	0	7
15.0-19.99	8	5	2	5	1	2	10	3	1	2	1	2
20.0-24.99	10	2	0	2	0	1	11	1	1	1	0	0
25.0-29.99	5	10	3	9	2	4	7	8	3	6	2	3
30.0-34.99	4	6	2	4	0	1	1	9	2	5	0	3
35.0-39.99	4	6	1	5	0	3	4	6	2	5	0	3
40.0-44.99	4	0	0	0	0	0	4	0	0	0	0	0
45.0-49.99	1	1	0	1	0	1	0	2	1	1	1	1
50.0-54.99	0	1	1	0	1	0	0	1	0	1	0	1
TOTAL	56	123	17	106	6	61	80	99	21	82	5	38

TABLE 20

FILE 1

15 MINUTE ANALYSIS SUMMARY

	RAYLEIGH DISTRIBUTION						FORRISTALL DISTRIBUTION					
SIG HEIGHT RANGE	# REC. PASSED	# REC. FAILED	ALL POINTS H	ALL POINTS L	G.T. 5% H	G.T. 5% L	# REC. PASSED	# REC. FAILED	ALL POINTS H	ALL POINTS L	G.T. 5% H	G.T. 5% L
0 -4.99	0	12	2	12	0	12	0	12	1	12	0	8
5.0-9.99	0	20	1	20	1	19	2	18	1	18	1	12
10.0-14.99	0	24	3	24	0	14	9	15	4	14	0	3
15.0-19.99	0	7	1	7	0	3	2	5	1	5	0	1
20.0-24.99	3	2	0	2	0	1	3	2	0	2	0	0
25.0-29.99	2	4	2	4	0	1	3	3	1	2	0	1
30.0-34.99	2	6	2	6	1	3	0	8	4	7	2	4
35.0-39.99	3	1	1	1	0	0	3	1	1	0	0	0
40.0-44.99	3	1	0	1	0	1	1	3	2	3	0	1
TOTAL	13	77	12	77	2	54	23	67	15	63	3	30

TABLE 21

FILE I

30 MINUTE ANALYSIS SUMMARY

	RAYLEIGH DISTRIBUTION						FORRISTALL DISTRIBUTION					
SIG HEIGHT RANGE	# REC. PASSED	# REC. FAILED	ALL POINTS		G.T. 5%		# REC. PASSED	# REC. FAILED	ALL POINTS		G.T. 5%	
			H	L	H	L			H	L	H	L
0-4.99	0	6	3	6	0	6	0	6	1	6	0	6
5.0-9.99	1	9	0	9	0	9	1	9	1	8	0	8
10.0-14.99	0	12	1	12	1	7	2	10	3	7	2	4
15.0-19.99	0	3	0	3	0	2	1	2	0	2	0	1
20.0-24.99	1	2	0	2	0	2	1	2	0	2	0	1
25.0-29.99	0	2	0	2	0	1	2	0	0	0	0	0
30.0-34.99	0	5	1	5	0	3	0	5	2	4	0	2
35.0-39.99	1	1	1	0	0	0	1	1	1	0	0	0
40.0-44.99	1	1	0	1	0	0	1	1	1	1	0	0
TOTAL	4	41	6	40	1	30	9	36	9	30	2	22

that the sample was Rayleigh at the 95% level. All samples went below the lower limit and one also went above. For the second 35, six pass, one went outside the upper bound only, two were both above and below, and 26 were below and so on for the complete table.

For the five full tables for 35 7.5 minute samples to summarize; for the first 35, 0 passes, 34 low and one both high-low; for the next 35, 6 passes, 26 low, one high, and 2 both high-low; for the next 35, 13 passes, 19 low, one high and 3 high-low; for the next 35, 21 passes, 12 low, and 2 both high-low, and for the last full 35, 15 passes, 14 low, 5 high and one both high-low. The final 5 records gave 2 passes, 2 low and one high. The totals for the 180 records were 56 passes, 107 low, 8 high and 9 high-low. Of the 124 that did not pass, there were 48 for which only 5% of the sample was outside of the bounds. With broader bounds to account for the lack of a random sample, with the use of a binomial pdf to define these bounds, and with the use of a somewhat reduced value for m_o, instead of 31% passing (or perhaps (56 + 48)/180 which equals 58%) quite a few might have passed.

The 15 minute samples can be interpreted in much the same way. The larger the sample the narrower the confidence bounds become and the more sensitive the interpretation to the effects of a non-random sample. Of the 90 samples, 13 out of 90 pass (14%) that could possibly be increased to (13 + 22)/90 or 39%. The first record to pass is record 64 suggesting that the possible error in finding m_o is larger (in percent) for the lower waves. Of the last 27 records, 48% pass. With 8 more having only 5% of the heights outside of the computed range, possibly 78% could have passed as a result of these effects.

There were 45 thirty minute samples. The effect of increased values for N, with the non-randomness of the sample narrowed the confidence interval so that only 4 samples, 9%, passed. The percentage of points outside of the confidence interval increased markedly with only 9 under 5% so that only 29% ((4 + 9)/45) either came close or passed.

The stratification of the data into significant height ranges shows a tendency for the high waves to fit the Rayleigh but not the lower waves. Slightly too high values for m_o for the lower waves in particular could

account for this effect. This possibility seems to be reinforced by Table 15.

The method of plotting the data does no affect the results except for the redefinition of the sample cdf. Note that the point for R equal to 1 is a tick on the horizontal axis. It has been marked more heavily on the copies of the figures from Spring. The same data could have been presented in a form similar to Figure 32 to 34. The results described by Spring in the material quoted closely parallel the predictions made about the properties of small samples in the preceding material of our study.

These results need to be contrasted to the results of Donelan and Pierson (1983). The data from the wind wave flume under known stationary conditions had the property that combining 16 samples yielded a far more smoothly varying spectrum with fairly narrow confidence bounds. Had ten times more data been obtained it is not difficult to postulate that the composite spectrum would have been smoother and that the confidence interval would have been reduced by a factor of $(10)^{\frac{1}{2}}$. For the Camille data, the combined effect of non stationarity and the lack of a random sample penalize large samples by imposing requirements on the presumed confidence bounds that are too strict.

SUMMARY

Data corresponding to eleven thousand successive crest to trough wave heights that required twenty one hours and forty minutes to pass a Baylor wave staff at an offshore oil platform were studied as fifty five sets of two hundred successive waves. The spectra for this sequence of waves generated during Hurricane Camille were fairly well hindcast (or specified) by the wave forecasting model described by Cardone, Pierson and Ward (1976). Significant heights from the spectra of the hindcast model compared favorably with significant heights from the spectra estimated from the wave time histories within the confidence intervals for the significant height.

At the start of the period, the significant heights were between 3 and 4 feet. Toward the end of the period, for the 200 wave samples, the significant heights exceeded 30 feet, and the last sample exceeded 40 feet. Individual waves over 50 feet were measured for record 48 (72.2 feet), record 50(65.4 feet), record 51(68.5 feet) and record 55(70.4 feet).

There is a widening gap between what is known about periodic nonlinear waves, periodic nonlinear breaking waves, the various attempts to model nonlinear irregular waves and theories for the interpretation of actual ocean wave time histories. Realistic periodic breaking waves can be created mathematically that are caused to break by, among other effects, applying a transient mathematical atmospheric pressure field (a gust of wind?) to the waves. Witham (1965) implies that exact analytical solutions of the equations of motion that describe real waves on the actual ocean are "out of the question".

The study of mechanically generated transient wave groups has produced realistic extreme waves and breaking waves that can be exactly predicted after the fact. The theories of Witham do not seem to have a consistent set of derived requirements, but they may be capable of extention, and correction, to account for random waves to third order.

Studies of waves actually generated by the wind have shown that irregularities in the coastline can produce directional wave spectra that have values appropriate to sources such as bays and coastline indentations. This implies that higher resolution model in space and time will be needed to describe waves. Higher order effects in wave spectra are detectable for bound second harmonics, but the corrections to the variance spectra are two orders of magnitude lower

than the first order peak. Corrections, according to these results, to the phase speeds of the first order components are small. If to the contrary, a Witham - type theory applies, a local-type phase speed also applies that is a function of the "local" wave height and "local" frequency. Although the averages may verify well, individual wave groups may travel at quite different speeds and group velocities.

The theoretical models that modify the Rayleigh pdf all begin with the linear wave model and all give results that reduce the probabilities of the higher waves compared to the Rayleigh. One new model theoretically attempts to account for wider band spectra such that a changing wave envelope results in a regression toward the middle values for both low and high waves. Another, of several, models breaking waves and concludes that there is a deficit of high waves compared to the Rayleigh model. The available models treat those aspect of random waves that always result in a lower frequency of occurrence of the higher waves. Higher order effects that would produce higher crest to trough heights, even at third order, are not included. The change in wave form as waves steepen to produce sharp crests with a 120° angle between front and back (or less) are not reproduced. It is not known whether or not these unaccounted-for effects could re-amplify the higher actual waves so as to cancel out the effects so far predicted by the linear models.

Verification of these models has consisted of a limited number of sample height distributions plotted against the theoretical distributions without tests of goodness-of-fit, the consideration of the required properties of a random sample and other complications having to do with the actual ways that waves are recorded.

The analysis of these fifty five samples of 200 consecutive waves has been restricted to the time domain in an attempt to reconcile the properties of sample distributions of crest to trough wave heights with the properties that would be predicted from spectral estimates such as the root mean square height, the mean height, the significant height, the average of the one tenth highest and the extreme waves in each sample. In so doing, an almost insurmountable difficulty arises. The sample of 200 waves is not a random sample.

Although the fact that the sample is not a random sample would be obvious from the properties of a stationary Gaussian process, it is not

obvious enough to raise a warning flag when testing whether or not a given sample fits a given model. In the time domain alone, the covariance function demonstrates the time dependence of nearby values, but the time (and space) dependence of successive waves is not also appreciated as a corollary of this property.

To test the independence of successive wave heights, the heights were modeled as a ten state one step Markov process. The model proved to be only approximate in that powers of the transitions matrix did not reproduce the two step and three step transitions. As a ten step Markov process, which allowed the data to be pooled into larger samples by partially removing trends, the results showed that the height of a passing wave was not a useful predictor of the height of the next wave. Any one of the ten states could follow any one of the initial ten states. There was, nevertheless, a weak, but obvious, dependence of the height of the following wave on the wave that had just passed. This dependence invalidates the assumption that the sample is a random sample.

In the spectral domain, there is the concept of an effective number of independent points which can be determined from the spectral estimate. For time correlated wave heights, no analogous way to determine the effective number of independent heights in the sample was found, but it is clear that the effective number is less than the actual number and that the effective number decreased for the waves in Camille as heights of the waves increased.

The available methods to test the goodness-of-fit of a samnle to a theory that are applicable to this problem are all asymptotically valid for large samples. A sample of 200 values, in fact less, is not a large sample, and so the tests can be misleading. For a small sample, there is a demonstrable tendency to miss the lower and higher values in the sample and to over sample the middle range of values. Combined with the smaller samples possible for a truncated distribution, this in turn leads to highly variable estimates of truncated moments that more often than not are too low due to chance alone.

The normalized cumulative density function for the highest wave in each of the 55 samples did not have the properties that would be predicted for a random sample of 200 waves, except for the six highest. When tested against

the theory for a random sample of either 200, or 100 waves, the highest wave in the sample is not unusual if the spectral moment is used.

CONCLUSIONS AND RECOMMENDATIONS

One possible conclusion for this investigation would be to claim "insufficient evidence" and that no conclusions are possible. Quatrain XXVII of the Rubaiyat of Omar Khayyam (Fitzgerald (undated)) might be one way for the reader to summarize this study of the waves that were measured during Hurricane Camille. However, the complicated interaction of numerous factors in real data can lead to multiple possible causes for the same observed effect. To focus on one or two of these factors can lead, and has lead, to misleading conclusions. The conclusions that can be drawn from this study must be highly qualified. The conclusions follow.

1) Samples of successive crest to trough ocean wave heights can be processed to give various statistics in a number of different ways. The significant wave height can be found nine different ways. Given the spectrum of the sample, the significant wave height can also be found from the area under the spectrum. Often the actual sample significant height is lower than one computed from any statistics based on the total sample and still lower than the one computed from the spectrum.

2) The tests of goodness-of-fit that compare the sample to the Rayleigh pdf usually conclude that the Rayleigh pdf does not describe the data.

3) The five possible reasons for these two observed facts are:

a) The tendency for the Rayleigh envelope to regress toward the mean during the full wave cycle if the spectrum is not narrow and thus to generate a pdf with a deficit of both low waves and high waves compared to the Rayleigh.

b) The breaking of the higher waves on the ocean thus causing a deficit of the higher waves, with this deficit being compensated for by the frequency of occurrence of intermediate wave heights.

c) The difficulties in drawing inferences from a small sample. The probability of drawing a random sample that will include the expected number of sample points from both the high end and the low end of the theoretical population is well under 50% so that many samples, by chance, will have the properties predicted by (a)

and (b) above.

d) The small samples that are available to test these models are not random samples. They are consequently, in effect, even smaller than the sample size indicates. The use of the size of the non-random sample in goodness-of-fit tests imposes an unduely severe restriction on the sampling variability, frequently yielding the rejection of the hypothesis being tested.

e) The area under the variance spectrum of the ocean wave time history may have been over estimated because of digitization errors.

4) It is not possible to separate the five reasons for what was observed with the present data set from Camille although this data is very high quality and did, in fact, accomplish the difficult goal of measuring very high waves.

5) The decreases in the spectral variance needed to bring about agreement with the wave height statistics in small. A more careful spectral analysis and an estimate of the contribution from various error sources to the total variance would be useful supplemental information for each published spectrum.

6) Reasons (a),(b),(c), and (d), with the effects for (e) removed if possible, would be indistinguishable one from the other for a small typical sample. The additional free parameters from these theories would always make it possible to fit many of the samples in a way that would appear to be an improvement. If the theoretical effects are actually present, fitting a sample by means of the additional free parameters may over-predict the effect being sought.

7) Nonlinear wave effects that have not been considered would operate in ways that would increase the heights of the higher waves in a sample. It is not known whether or not, and by how much, these effects would cancel out the effects predicted in (a) and (b) above.

8) The case for other theoretical probability density functions that appear to be an improvement on the Rayleigh pdf has neither been proven nor disproven on the basis of either the present literature or this analysis of the Camille data.

9) More carefully estimated spectra, with due consideration of digitization errors and other sources of unwanted noise, can probably provide a reliable enough estimate of E_o (or m_o) to permit the use of Rayleigh statistics.

10) Apart from frequency dependent instrument calibration errors, remaining digitization errors will make the estimate of the variance too high, and for design purposes the estimates will be conservative because they will be too high.

11) If the only available data consist of the crest to trough heights from the original time history so that spectra cannot be estimated, it might be advisable to compute E_o both from the mean and the maximum likelihood estimate and to use the larger of the two to compute the significant wave height. Bounds on the significant height and the average of the one tenth highest as in equations (128) and (129) with a correction by means of a guess of the effective sample size (tentatively one half for seas and even less for swell) would, at least, suggest that these numbers are not well known.

12) Exceptionally high waves can and do happen. They were not particularly unusual, or improbable, in the context of the eleven thousand waves in the total sample for Hurricane Camille. However, these exceptional waves must be very nonlinear and present theories are inadequate to describe them in the context of a non-stationary random process.

13) Both the Rayleigh pdf and extreme wave theory must obviously fail if tested against an extremly long duration stationary random sample of waves. Those heights that are not possible from physical-hydrodynamic considerations are not presently known and were not revealed by this study.

From the above conclusions, some recommendations can be made. These recommendations follow.

1) Spectra estimated from ocean wave time histories need to be checked for digitization errors and other sources of error. After correction, the area under the spectrum, and its square root, to calculate other wave statistics should be used.

2) Although not treated in this report, it is known that different

organizations use different spectral modification procedures. Measurement and data analysis systems need to be intercompared and standardized.

3) Extreme waves can be deliberately modeled in such a way that they will appear at a given place at a given time, even perhaps as part of a random sea. They may not produce the worst possible condition for a particular model design, but the conditions produced, especially if the carrier frequency and the Fourier integral spectrum are varied, will provide useful test data on extreme conditions. The chance that the design will be successful under real conditions will be increased.

4) Random seas can be generated in wind wave flumes under truly stationary conditions that can be made to last as long as desired. The sampling variability of sub-samples is understood in terms of spectra. A very large sample as in the work of Huang and Long (1980), except in larger facilities, could eliminate small sample effects and provide results that could, perhaps, be scaled up to actual conditions.

5) Ocean wave forecasts are based on forecasting the spectra and ought to be verified against spectra. A convenient linear metric for comparing wave forecasts, and wave climatologies based on past wind data, is four times the square root of the area under the spectrum. Such a metric is the first gross check of how good the model is. Heights that are greatly in error show inadequacies of the model. Present errors in wave forecasting models exceed the kinds of differences found in the various ways to compute truncated moments and in the corrections that would be implied if any of the newer probability models were used. It would be necessary to predict spectra accurate enough to specify at least the first and second moments of the spectra correctly before these new models could be used.

6) At present for long crested random waves only, nonlinear models suggest that as an advancing group of waves coalesce to form a very high wave, the "length" of the waves in the group increases so as to preserve the frequency content of the spectrum. The linear theory for ship motions in waves involves the frequency of encounter and integrals of the wave form along the hull of the ship. If the length of the apparent wave along the hull of the ship is incorrect because of nonlinear effects, the ship response will not be correctly computed.

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We appreciate the kindness of Prof. M. Aziz Tayfun in providing additional details on his forthcoming publications, which have now appeared, and computor program listings for possible further tests of his results. Prof. M. S. Longuit-Higgins also provided manuscript copies of forthcoming publications and reprints of the papers cited herein.

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APPENDIX A

STATISTICS ON SAMPLES

1 TO 55

3 1	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	3.8				3.0		1.9				0.8	4.4	4.9	5.2	6.1
H10H		3.9			3.1		1.9				0.8	4.4	5.0	5.3	6.2
H10M			4.0		3.2		2.0				0.8	4.6	5.2	5.4	6.4
HSIP		4.0		3.1			2.0				0.8	4.6	5.1	5.4	6.4
HSIG		4.1			3.2		2.0				0.8	4.7	5.2	5.5	6.5
HSIM		4.2				3.3	2.0				0.8	4.8	5.4	5.7	6.7
HBAR		4.3			3.4		2.1				0.8	4.9	5.5	5.8	6.8
HMLE		4.2			3.3			2.0			0.8	4.7	5.3	5.6	6.6
HCDF		4.2			3.3				2.1		0.8	4.8	5.4	5.7	6.8
HSPT		4.6			3.7					2.3	0.9	5.3	6.0	6.3	7.4
H200														4.8	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.309	2.347	2.421	1.888	1.933	1.979	1.266	2.895
NORM HBAR	2.285	2.323	2.396	1.869	1.913	1.959	1.253	2.865
NORM HMLE	2.349	2.388	2.463	1.921	1.966	2.014	1.288	2.945

7 2	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	3.7				2.0		1.8				0.7	4.2	4.7	5.0	5.8
H10H		3.7			2.0		1.8				0.7	4.3	4.8	5.0	6.0
H10M			3.9		3.0		1.9				0.8	4.4	4.9	5.2	6.2
HSIP		3.9		3.1			1.9				0.8	4.4	5.0	5.3	6.2
HSIG		4.0			3.1		2.0				0.8	4.5	5.1	5.4	6.3
HSIM		4.1				3.2	2.0				0.8	4.6	5.2	5.5	6.5
HBAR		4.2			3.3		2.1				0.8	4.8	5.4	5.7	6.8
HMLE		4.1			3.2			2.3			0.8	4.7	5.3	5.6	6.6
HCDF		4.3			3.4				2.1		0.9	4.9	5.6	5.9	6.9
HSPT		4.6			3.6					2.3	0.9	5.2	5.9	6.2	7.3
H200														4.5	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.150	2.191	2.264	1.796	1.835	1.875	1.227	2.629
NORM HBAR	2.196	2.237	2.312	1.834	1.874	1.914	1.253	2.685
NORM HMLE	2.261	2.304	2.380	1.888	1.929	1.971	1.291	2.764

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3	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	3.7				2.0		1.8				0.7	4.3	4.8	5.1	6.0
H10H		3.8			3.0		1.9				0.7	4.4	4.9	5.2	6.1
H10M			3.9		3.1		1.9				0.8	4.5	5.1	5.3	6.3
HSIP		3.9		3.1			1.9				0.8	4.5	5.1	5.3	6.3
HSIG		4.0			3.2		2.0				0.8	4.6	5.2	5.4	6.4
HSIM		4.1				3.2	2.0				0.8	4.7	5.3	5.6	6.6
HBAR		4.3			3.4		2.1				0.8	4.9	5.5	5.8	6.8
HMLE		4.2			3.3			2.1			0.8	4.8	5.3	5.6	6.7
HCDF		4.4			3.4				2.2		0.9	5.0	5.6	5.9	7.0
HSPT		4.6			3.6					2.3	0.9	5.2	5.9	6.2	7.3
H200														4.6	

		H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM	HCDF	2.181	2.222	2.294	1.804	1.841	1.883	1.227	2.680
NORM	HBAR	2.229	2.271	2.345	1.844	1.882	1.924	1.253	2.739
NORM	HMLE	2.288	2.331	2.407	1.892	1.932	1.975	1.287	2.811

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4	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	4.3				3.4		2.1				0.8	4.9	5.5	5.8	6.9
H10H		4.4			3.4		2.2				0.9	5.0	5.6	5.9	7.0
H10M			4.5		3.5		2.2				0.9	5.1	5.8	6.1	7.2
HSIP		4.6		3.6			2.3				0.9	5.3	5.9	6.2	7.4
HSIG		4.7			3.7		2.3				0.9	5.4	6.0	6.4	7.5
HSIM		4.8				3.8	2.3				0.9	5.5	6.2	6.5	7.7
HBAR		5.0			3.9		2.5				1.0	5.7	6.4	6.8	8.0
HMLE		4.9			3.8			2.4			1.0	5.5	6.2	6.6	7.7
HCDF		5.1			4.0				2.5		1.0	5.8	6.5	6.9	8.1
HSPT		5.3			4.2					2.6	1.0	6.0	6.8	7.1	8.4
H200														5.3	

		H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM	HCDF	2.167	2.201	2.263	1.821	1.862	1.903	1.238	2.678
NORM	HBAR	2.195	2.229	2.292	1.844	1.885	1.927	1.253	2.712
NORM	HMLE	2.264	2.299	2.364	1.903	1.945	1.988	1.293	2.797

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5	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	4.4				3.5		2.2				0.9	5.0	5.7	6.0	7.0
H10H		4.5			3.5		2.2				0.9	5.1	5.7	6.1	7.1
H10M			4.6		3.6		2.3				0.9	5.3	5.9	6.3	7.4
HSIP		4.7		3.7			2.3				0.9	5.3	6.0	6.3	7.5
HSIG		4.8			3.8		2.4				0.9	5.5	6.1	6.5	7.6
HSIM		4.9				3.9	2.4				1.0	5.6	6.3	6.6	7.8
HBAR		4.9			3.8		2.4				1.0	5.5	6.2	6.6	7.8
HMLE		4.8			3.8			2.4			0.9	5.5	6.2	6.5	7.7
HCDF		4.9			3.9				2.4		1.0	5.6	6.3	6.7	7.9
HSPT		5.5			4.3					2.7	1.1	6.3	7.1	7.5	8.8
H200														5.7	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.281	2.316	2.393	1.900	1.949	1.997	1.239	2.942
NORM HBAR	2.308	2.344	2.422	1.923	1.972	2.021	1.253	2.978
NORM HMLE	2.336	2.372	2.450	1.946	1.996	2.045	1.268	3.013

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6	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	4.5				3.6		2.2				0.9	5.2	5.8	6.1	7.2
H10H		4.6			3.6		2.3				0.9	5.2	5.9	6.2	7.3
H10M			4.7		3.7		2.3				0.9	5.4	6.0	6.4	7.5
HSIP		4.8		3.8			2.4				1.0	5.5	6.2	6.6	7.7
HSIG		4.9			3.9		2.4				1.0	5.6	6.3	6.7	7.9
HSIM		5.1				4.0	2.4				1.0	5.8	6.5	6.8	8.1
HBAR		5.3			4.1		2.6				1.0	6.0	6.7	7.1	8.4
HMLE		5.1			4.0			2.5			1.0	5.8	6.6	6.9	8.2
HCDF		5.4			4.2				2.6		1.1	6.1	6.9	7.3	8.6
HSPT		5.7			4.5					2.8	1.1	6.5	7.3	7.7	9.1
H200														5.8	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.143	2.172	2.227	1.806	1.843	1.882	1.227	2.768
NORM HBAR	2.189	2.219	2.275	1.845	1.882	1.923	1.253	2.828
NORM HMLE	2.253	2.284	2.341	1.899	1.937	1.979	1.299	2.910

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7	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	4.5				3.5		2.2				0.9	5.1	5.7	6.0	7.1
H10H		4.5			3.6		2.2				0.9	5.2	5.8	6.1	7.2
H10M			4.7		3.7		2.3				0.9	5.3	6.0	6.3	7.4
HSIP		4.8		3.8			2.4				0.9	5.5	6.2	6.5	7.7
HSIG		4.9			3.9		2.4				1.0	5.6	6.3	6.6	7.8
HSIM		5.0				3.9	2.4				1.0	5.7	6.4	6.8	8.0
HBAR		5.3			4.2		2.6				1.0	6.0	6.8	7.1	8.4
HMLE		5.1			4.0			2.5			1.0	5.8	6.5	6.9	8.1
HCDF		5.4			4.2				2.6		1.1	6.1	6.9	7.2	8.6
HSPT		5.8			4.6					2.9	1.1	6.7	7.5	7.9	9.3
H200														5.9	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.120	2.151	2.217	1.802	1.836	1.873	1.235	2.822
NORM HBAR	2.152	2.183	2.250	1.928	1.863	1.901	1.253	2.840
NORM HMLE	2.225	2.258	2.327	1.891	1.927	1.966	1.296	2.941

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8	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	F0	H 5	HMP	H200	H 95
H10P	5.1				4.0		2.5				1.0	5.8	6.6	6.9	8.2
H10H		5.2			4.1		2.6				1.0	5.9	6.7	7.0	8.3
H10M			5.4		4.2		2.7				1.1	6.1	6.9	7.3	8.6
HSIP		5.5		4.3			2.7				1.1	6.3	7.0	7.4	8.8
HSIG		5.6			4.0		2.8				1.1	6.4	7.2	7.6	9.0
HSIM		5.8				4.5	2.8				1.1	6.6	7.4	7.8	9.2
HBAR		5.9			4.6		2.9				1.2	6.7	7.6	8.0	9.4
HMLE		5.7			4.5			2.8			1.1	6.5	7.3	7.8	9.2
HCDF		5.9			4.7				2.9		1.2	6.8	7.6	8.0	9.5
HSPT		6.3			5.0					3.1	1.2	7.2	8.1	8.6	10.1
H200														6.1	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.196	2.235	2.313	1.853	1.898	1.940	1.248	2.607
NORM HBAR	2.206	2.245	2.323	1.861	1.906	1.948	1.253	2.617
NORM HMLE	2.274	2.314	2.395	1.919	1.965	2.008	1.292	2.698

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9	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	5.4				4.2		2.6				1.1	6.1	6.9	7.3	8.6
H10H		5.5			4.3		2.7				1.1	6.2	7.0	7.4	8.7
H10M			5.7		4.5		2.8				1.1	6.5	7.3	7.7	9.1
HSIP		5.5		4.3			2.7				1.1	6.3	7.0	7.4	8.8
HSIG		5.6			4.0		2.8				1.1	6.3	7.2	7.6	9.0
HSIM		5.8				4.5	2.9				1.1	6.6	7.4	7.8	9.2
HBAR		5.8			4.6		2.9				1.1	6.7	7.5	7.9	9.3
HMLE		5.7			4.5			2.9			1.1	6.5	7.4	7.8	9.2
HCDF		5.9			4.7				2.9		1.2	6.7	7.6	8.0	9.4
HSPT		6.3			5.0					3.1	1.2	7.2	8.1	8.6	10.1
H200														6.4	

		H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM	HCDF	2.313	2.358	2.441	1.857	1.900	1.946	1.237	2.770
NORM	HBAR	2.344	2.389	2.473	1.881	1.925	1.972	1.253	2.806
NORM	HMLE	2.384	2.433	2.515	1.914	1.958	2.006	1.275	2.854

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10	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	6.0				4.7		2.9				1.2	6.8	7.7	8.1	9.5
H10H		6.1			4.8		3.0				1.2	6.9	7.8	8.2	9.7
H10M			6.3		5.0		3.1				1.2	7.2	8.1	8.5	10.1
HSIP		6.1		4.8			3.0				1.2	6.9	7.8	8.2	9.7
HSIG		6.2			4.9		3.1				1.2	7.1	8.0	8.4	9.9
HSIM		6.4				5.0	3.1				1.3	7.3	8.2	8.7	10.2
HBAR		6.3			5.0		3.1				1.2	7.2	8.1	8.6	10.1
HMLE		6.3			4.9			3.1			1.2	7.2	8.0	8.5	10.0
HCDF		6.3			5.0				3.1		1.2	7.2	8.1	8.6	10.1
HSPT		6.8			5.3					3.3	1.3	7.7	8.7	9.1	10.8
H200														7.7	

		H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM	HCDF	2.406	2.448	2.537	1.924	1.972	2.024	1.254	3.086
NORM	HBAR	2.424	2.446	2.534	1.922	1.970	2.022	1.253	3.084
NORM	HMLE	2.425	2.468	2.557	1.939	1.988	2.040	1.265	3.111

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11	H12P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H12P	6.9				5.5		3.4				1.4	7.9	8.9	9.4	11.1
H10H		7.0			5.5		3.5				1.4	8.0	9.0	9.5	11.2
H10M			7.2		5.7		3.6				1.4	8.2	9.2	9.8	11.5
HSIP		7.3		5.7			3.6				1.4	8.3	9.4	9.9	11.7
HSIG		7.5			5.9		3.7				1.5	8.5	9.6	10.1	11.9
HSIM		7.6				6.0	3.7				1.5	8.7	9.8	10.3	12.2
HBAR		7.9			6.2		3.9				1.5	9.0	10.1	10.6	12.5
HMLE		7.7			6.0			3.8			1.5	8.7	9.8	10.3	12.2
HCDF		7.9			6.2				3.9		1.6	9.1	10.2	10.7	12.7
HSPT		8.5			6.7					4.2	1.7	9.7	11.0	11.6	13.6
H200														9.0	

	H12P	H1CH	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.224	2.257	2.314	1.841	1.883	1.928	1.240	2.897
NORM HBAR	2.248	2.280	2.338	1.860	1.903	1.948	1.253	2.928
NORM HMLE	2.308	2.342	2.401	1.919	1.954	2.000	1.287	3.007

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12	H12P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H12P	6.8				5.3		3.3				1.3	7.7	8.7	9.2	10.8
H10H		6.9			5.0		3.4				1.4	7.9	8.8	9.5	11.0
H10M			7.1		5.6		3.5				1.4	8.1	9.1	9.6	11.3
HSIP		7.1		5.6			3.5				1.4	8.1	9.1	9.6	11.3
HSIG		7.3			5.7		3.6				1.4	8.3	9.3	9.8	11.6
HSIM		7.4				5.8	3.6				1.5	8.5	9.5	10.0	11.8
HBAR		7.5			5.9		3.7				1.5	8.5	9.6	10.1	11.9
HMLE		7.4			5.8			3.6			1.4	8.4	9.4	10.0	11.7
HCDF		7.6			6.0				3.7		1.5	8.7	9.7	10.3	12.1
HSPT		8.6			6.7					4.2	1.7	9.8	11.0	11.6	13.7
H200														8.2	

	H12P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.276	2.312	2.383	1.870	1.913	1.956	1.233	2.753
NORM HBAR	2.314	2.351	2.423	1.901	1.945	1.989	1.253	2.800
NORM HMLE	2.349	2.386	2.459	1.929	1.974	2.019	1.272	2.841

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13	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	7.5				5.0		3.7				1.5	8.6	9.6	10.2	12.0
H10H		7.7			6.1		3.8				1.5	8.8	9.9	10.4	12.3
H10M			8.2		6.0		4.0				1.6	9.3	10.5	11.0	13.0
HSIP		7.5		5.9			3.7				1.5	8.5	9.6	10.1	11.9
HSIG		7.7			6.0		3.8				1.5	8.7	9.8	10.4	12.2
HSIM		7.9				6.2	3.8				1.5	9.0	10.1	10.7	12.6
HBAR		8.0			6.3		3.9				1.6	9.1	10.2	10.8	12.7
HMLE		7.9			6.2			3.9			1.5	9.0	10.1	10.6	12.6
HCDF		8.0			6.3				3.9		1.6	9.1	10.2	10.8	12.7
HSPT		8.9			6.0					4.3	1.7	10.1	11.3	11.9	14.1
H200														11.0	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.426	2.468	2.608	1.881	1.930	1.984	1.252	3.507
NORM HBAR	2.428	2.471	2.610	1.883	1.931	1.985	1.253	3.510
NORM HMLE	2.433	2.497	2.638	1.903	1.952	2.006	1.267	3.547

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14	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	8.2				6.0		4.0				1.6	9.3	10.5	11.0	13.0
H10H		8.3			6.5		4.1				1.6	9.5	10.6	11.2	13.2
H10M			8.6		6.7		4.2				1.7	9.8	11.0	11.6	13.7
HSIP		8.3		6.6			4.1				1.6	9.5	10.7	11.3	13.3
HSIG		8.6			6.7		4.2				1.7	9.8	11.0	11.6	13.7
HSIM		8.8				6.9	4.2				1.7	10.1	11.3	11.9	14.1
HBAR		8.5			6.7		4.2				1.7	9.7	10.9	11.5	13.6
HMLE		8.5			6.7			4.2			1.7	9.7	10.9	11.5	13.6
HCDF		8.5			6.7				4.2		1.7	9.7	10.9	11.5	13.6
HSPT		9.4			7.0					4.6	1.8	10.7	12.0	12.7	15.0
H200														13.3	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.444	2.482	2.563	1.961	2.015	2.073	1.252	3.988
NORM HBAR	2.446	2.485	2.566	1.963	2.017	2.075	1.253	3.992
NORM HMLE	2.437	2.476	2.556	1.954	2.009	2.068	1.249	3.977

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15	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	8.0				6.3		3.9				1.6	9.1	10.3	10.8	12.9
H10H		8.1			6.6		4.0				1.6	9.3	10.4	11.0	13.0
H10M			8.3		6.5		4.1				1.6	9.5	10.7	11.3	13.3
HSIP		8.6		6.7			4.2				1.7	9.8	11.0	11.6	13.7
HSIG		8.8			6.0		4.3				1.7	10.0	11.3	11.9	14.0
HSIM		9.0				7.1	4.3				1.8	10.3	11.6	12.2	14.4
HBAR		9.2			7.0		4.5				1.8	10.5	11.8	12.4	14.7
HMLE		8.9			7.0			4.4			1.8	10.2	11.5	12.1	14.3
HCDF		9.3			7.3				4.6		1.3	10.6	11.9	12.6	14.8
HSPT		10.5			8.2					5.2	2.1	11.9	13.4	14.2	16.7
H200														9.2	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.190	2.227	2.282	1.846	1.893	1.942	1.241	2.518
NORM HBAR	2.217	2.251	2.305	1.865	1.912	1.962	1.253	2.544
NORM HMLE	2.278	2.313	2.370	1.917	1.966	2.017	1.288	2.615

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16	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	8.6				6.7		4.2				1.7	9.8	11.0	11.6	13.7
H10H		8.8			6.0		4.3				1.7	10.0	11.2	11.8	14.0
H10M			9.1		7.1		4.5				1.8	10.3	11.6	12.3	14.5
HSIP		8.8		6.9			4.3				1.7	10.0	11.2	11.8	14.0
HSIG		9.0			7.1		4.4				1.8	10.2	11.5	12.1	14.3
HSIM		9.2				7.2	4.4				1.8	10.5	11.8	12.5	14.7
HBAR		9.0			7.1		4.5				1.8	10.3	11.6	12.2	14.4
HMLE		9.0			7.1			4.4			1.8	10.3	11.5	12.2	14.4
HCDF		9.0			7.1				4.4		1.8	10.3	11.5	12.2	14.4
HSPT		10.0			7.0					4.9	2.0	11.4	12.8	13.5	15.9
H200														11.8	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.425	2.476	2.567	1.947	1.996	2.049	1.258	3.340
NORM HBAR	2.416	2.467	2.557	1.940	1.988	2.041	1.253	3.327
NORM HMLE	2.429	2.480	2.571	1.951	1.999	2.052	1.260	3.346

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17	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	8.7				6.8		4.3				1.7	9.9	11.2	11.3	13.9
H10H		8.9			7.0		4.4				1.7	10.1	11.4	12.0	14.2
H10M			9.2		7.3		4.5				1.8	10.5	11.8	12.5	14.7
HSIP		9.0		7.0			4.4				1.8	10.2	11.5	12.1	14.3
HSIG		9.2			7.2		4.5				1.8	10.5	11.9	12.4	14.6
HSIM		9.4				7.4	4.5				1.8	10.7	12.1	12.7	15.0
HBAR		9.4			7.4		4.6				1.8	10.7	12.1	12.7	15.0
HMLE		9.3			7.3			4.5			1.8	10.6	11.9	12.6	14.9
HCDF		9.6			7.5				4.7		1.9	10.9	12.3	12.9	15.3
HSPT		10.5			8.2					5.2	2.1	11.9	13.4	14.2	16.7
H200														11.6	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.318	2.369	2.458	1.875	1.921	1.972	1.234	3.086
NORM HBAR	2.355	2.406	2.498	1.905	1.952	2.004	1.253	3.135
NORM HMLE	2.378	2.429	2.521	1.923	1.979	2.023	1.265	3.165

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18

18	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	9.4				7.4		4.6				1.8	10.7	12.1	12.7	15.0
H10H		9.7			7.6		4.8				1.9	11.0	12.4	13.1	15.4
H10M			10.2		8.0		5.0				2.0	11.6	13.1	13.8	16.3
HSIP		9.4		7.4			4.6				1.9	10.7	12.1	12.7	15.0
HSIG		9.7			7.6		4.8				1.9	11.0	12.4	13.1	15.4
HSIM		9.9				7.8	4.8				2.0	11.3	12.7	13.8	15.9
HBAR		9.6			7.6		4.7				1.9	11.0	12.3	13.0	15.4
HMLE		9.7			7.6			4.8			1.9	11.0	12.4	13.1	15.4
HCDF		9.5			7.5				4.7		1.9	10.8	12.2	12.8	15.1
HSPT		10.8			8.5					5.3	2.1	12.3	13.9	14.6	17.3
H200														14.7	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.528	2.594	2.738	1.989	2.041	2.098	1.272	3.988
NORM HBAR	2.491	2.557	2.699	1.960	2.012	2.068	1.253	3.887
NORM HMLE	2.480	2.545	2.686	1.951	2.002	2.058	1.248	3.869

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19	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	9.9				7.9		4.9				2.0	11.3	12.7	13.5	15.9
H10H		10.2			8.0		5.0				2.0	11.6	13.0	13.7	16.2
H10M			10.6		8.3		5.2				2.1	12.1	13.6	14.3	16.9
HSIP		10.1		8.0			5.0				2.0	11.6	13.0	13.7	16.2
HSIG		10.4			8.2		5.1				2.0	11.9	13.3	14.1	16.6
HSIM		10.7				8.4	5.1				2.1	12.2	13.7	14.5	17.1
HBAR		10.2			8.0		5.0				2.0	11.6	13.0	13.7	16.2
HMLE		10.3			8.1			5.1			2.0	11.7	13.2	13.9	16.4
HCDF		10.1			8.0				5.0		2.0	11.6	13.0	13.7	16.2
HSPT		11.9			9.4					5.9	2.3	13.6	15.3	16.1	19.0
H200														14.6	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.498	2.549	2.660	2.003	2.055	2.111	1.255	3.677
NORM HBAR	2.495	2.546	2.656	2.000	2.053	2.108	1.253	3.673
NORM HMLE	2.461	2.512	2.620	1.973	2.025	2.080	1.237	3.623

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20	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	10.6				8.3		5.2				2.1	12.1	13.6	14.3	16.9
H10H		10.8			8.5		5.3				2.1	12.3	13.8	14.6	17.2
H10M			11.1		8.7		5.5				2.2	12.7	14.3	15.0	17.8
HSIP		11.1		8.8			5.5				2.2	12.7	14.3	15.1	17.8
HSIG		11.4			8.0		5.6				2.2	13.0	14.6	15.4	18.2
HSIM		11.6				9.2	5.6				2.3	13.3	14.9	15.7	18.6
HBAR		12.1			9.5		6.0				2.4	13.8	15.5	16.4	19.3
HMLE		11.7			9.2			5.8			2.3	13.4	15.0	15.9	18.7
HCDF		12.2			9.6				6.0		2.4	13.9	15.6	16.5	19.5
HSPT		12.3			9.7					6.1	2.4	14.1	15.8	16.7	19.7
H200														13.1	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.213	2.253	2.324	1.828	1.869	1.913	1.243	2.732
NORM HBAR	2.231	2.271	2.342	1.843	1.884	1.928	1.253	2.754
NORM HMLE	2.300	2.341	2.415	1.900	1.943	1.988	1.292	2.839

A3

21	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	11.4				9.0		5.6				2.2	13.0	14.6	15.4	18.2
H10H		11.7			9.2		5.7				2.3	13.3	14.9	15.8	18.6
H10M			12.2		9.6		6.0				2.4	13.9	15.6	16.4	19.4
HSIP		11.4		8.9			5.6				2.2	13.0	14.6	15.4	18.2
HSIG		11.7			9.2		5.7				2.3	13.3	14.9	15.8	18.6
HSIM		12.0				9.4	5.7				2.3	13.6	15.3	16.2	19.1
HBAR		12.0			9.5		5.9				2.4	13.7	15.4	16.3	19.2
HMLE		11.8			9.3			5.8			2.3	13.5	15.2	16.0	18.9
HCDF		11.9			9.4				5.9		2.3	13.6	15.2	16.1	19.0
HSPT		12.8			10.1					6.3	2.5	14.6	16.4	17.3	20.5
H200														15.3	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.438	2.496	2.602	1.915	1.963	2.015	1.266	3.284
NORM HBAR	2.414	2.471	2.575	1.895	1.943	1.994	1.253	3.251
NORM HMLE	2.449	2.507	2.614	1.923	1.972	2.024	1.272	3.299

A7

22	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	12.0				9.7		6.1				2.4	14.1	15.9	16.8	19.8
H10H		12.7			9.9		6.2				2.5	14.4	16.2	17.1	20.2
H10M			13.1		10.3		6.5				2.6	15.0	16.8	17.7	20.9
HSIP		12.3		9.6			6.0				2.4	14.0	15.7	16.6	19.6
HSIG		12.6			9.9		6.2				2.5	14.4	16.2	17.1	20.1
HSIM		13.0				10.2	6.2				2.6	14.8	16.7	17.6	20.8
HBAR		12.4			9.7		6.1				2.4	14.1	15.9	16.8	19.8
HMLE		12.5			9.8			6.2			2.5	14.3	16.0	16.9	20.0
HCDF		12.3			9.7				6.1		2.4	14.0	15.8	16.7	19.7
HSPT		13.8			10.9					6.8	2.7	15.8	17.7	18.7	22.1
H200														15.3	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.565	2.618	2.713	1.993	2.053	2.118	1.262	3.163
NORM HBAR	2.548	2.600	2.695	1.979	2.039	2.104	1.253	3.142
NORM HMLE	2.526	2.578	2.672	1.962	2.021	2.086	1.243	3.110

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25	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HHP	H200	H 95
H10P	12.4				9.7		6.1				2.1	14.1	15.8	16.7	19.7
H10H		12.5			9.9		6.2				2.5	14.3	16.1	17.0	20.0
H10M			12.9		10.1		6.3				2.5	14.7	16.5	17.4	20.5
HSIP		12.8		10.1			6.3				2.5	14.6	16.5	17.4	20.5
HSIG		13.2			10.4		6.5				2.6	15.0	16.9	17.8	21.0
HSIM		13.5				10.6	6.5				2.6	15.4	17.3	18.3	21.5
HBAR		12.9			10.1		6.3				2.5	14.7	16.5	17.4	20.5
HMLE		13.0			10.2			6.4			2.5	14.8	16.6	17.5	20.7
HCDF		12.9			10.2				6.4		2.5	14.7	16.6	17.5	20.6
HSPT		14.2			11.1					7.0	2.8	16.2	18.2	19.2	22.6
H200														14.9	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.434	2.471	2.536	1.988	2.039	2.090	1.247	2.936
NORM HBAR	2.447	2.484	2.549	1.999	2.049	2.131	1.253	2.951
NORM HMLE	2.430	2.467	2.531	1.985	2.035	2.087	1.245	2.931

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24	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HHP	H200	H 95
H10P	11.6				9.1		5.7				2.3	13.2	14.9	15.7	18.5
H10H		11.8			9.3		5.8				2.3	13.5	15.1	16.0	18.8
H10M			12.2		9.6		6.0				2.4	14.0	15.7	16.6	19.5
HSIP		12.1		9.5			6.0				2.4	13.8	15.5	16.4	19.4
HSIG		12.4			9.8		6.1				2.4	14.1	15.9	16.8	19.8
HSIM		12.7				10.0	6.1				2.5	14.5	16.3	17.2	20.3
HBAR		13.0			10.2		6.4				2.6	14.8	16.6	17.6	20.7
HMLE		12.8			10.0			6.3			2.5	14.5	16.3	17.3	20.4
HCDF		13.2			10.4				6.5		2.6	15.1	16.9	17.9	21.1
HSPT		13.7			10.7					6.7	2.7	15.6	17.5	18.5	21.8
H200														17.0	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.234	2.274	2.359	1.837	1.879	1.922	1.231	3.268
NORM HBAR	2.273	2.314	2.401	1.870	1.912	1.956	1.253	3.326
NORM HMLE	2.315	2.357	2.445	1.904	1.947	1.992	1.276	3.387

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25	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	11.1				8.7		5.4				2.2	12.6	14.2	15.0	17.6
H10H		11.3			8.0		5.5				2.2	12.8	14.4	15.2	18.0
H10M			11.7		9.2		5.7				2.3	13.3	14.9	15.6	18.6
HSIP		11.2		8.8			5.5				2.2	12.7	14.3	15.1	17.8
HSIG		11.4			9.0		5.4				2.2	13.1	14.7	15.5	18.3
HSIM		11.8				9.2	5.6				2.3	13.4	15.1	15.9	18.8
HBAR		11.9			9.3		5.8				2.3	13.5	15.2	16.1	19.0
HMLE		11.7			9.2			5.8			2.3	13.4	15.0	15.8	18.7
HCDF		12.0			9.4				5.9		2.3	13.6	15.3	16.2	19.1
HSPT		13.6			10.7					6.7	2.7	15.5	17.4	18.4	21.7
H200														14.4	

		H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM	HCDF	2.354	2.399	2.482	1.871	1.917	1.968	1.245	3.063
NORM	HBAR	2.370	2.415	2.499	1.883	1.930	1.981	1.253	3.088
NORM	HMLE	2.404	2.453	2.535	1.910	1.957	2.010	1.271	3.128

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26	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	11.7				9.2		5.7				2.3	13.3	15.0	15.8	19.6
H10H		11.9			9.4		5.9				2.3	13.6	15.3	16.1	19.0
H10M			12.4		9.7		6.1				2.4	14.1	15.9	16.7	19.7
HSIP		12.0		9.5			5.9				2.4	13.7	15.4	16.3	19.2
HSIG		12.3			9.7		6.1				2.4	14.0	15.8	16.6	19.6
HSIM		12.6				9.9	6.1				2.5	14.4	16.2	17.1	20.1
HBAR		12.7			10.0		6.2				2.5	14.5	16.3	17.2	20.3
HMLE		12.5			9.9			6.2			2.5	14.3	16.1	17.0	20.0
HCDF		12.9			10.1				6.3		2.5	14.7	16.5	17.4	20.6
HSPT		14.0			11.0					6.9	2.7	15.9	17.9	18.9	22.3
H200														16.7	

		H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM	HCDF	2.306	2.351	2.443	1.866	1.911	1.959	1.233	3.288
NORM	HBAR	2.344	2.389	2.483	1.897	1.943	1.991	1.253	3.342
NORM	HMLE	2.372	2.418	2.513	1.920	1.966	2.015	1.269	3.383

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27	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	12.1				9.7		6.1				2.4	14.1	15.9	16.8	19.8
H10H		12.7			10.0		6.2				2.5	14.5	16.3	17.2	20.3
H10M			13.3		10.5		6.5				2.6	15.2	17.1	18.0	21.2
HSIP		12.9		10.2			6.4				2.5	14.7	16.6	17.5	20.6
HSIG		13.2			10.4		6.5				2.6	15.0	16.9	17.8	21.1
HSIM		13.5				10.6	6.5				2.6	15.4	17.3	18.2	21.5
HBAR		14.2			11.0		6.9				2.7	16.0	17.9	18.9	22.3
HMLE		13.7			10.7			6.7			2.7	15.6	17.5	18.5	21.9
HCDF		14.1			11.1				6.9		2.8	16.1	18.1	19.1	22.5
HSPT		14.4			11.3					7.1	2.8	16.4	18.4	19.4	22.9
H200														17.4	

		H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM	HCDF	2.238	2.291	2.403	1.835	1.873	1.915	1.244	3.147
NORM	HBAR	2.256	2.309	2.422	1.850	1.888	1.930	1.253	3.172
NORM	HMLE	2.312	2.364	2.480	1.894	1.933	1.976	1.283	3.247

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28	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	13.2				10.0		6.5				2.6	15.1	16.9	17.9	21.1
H10H		13.4			10.6		6.6				2.6	15.3	17.2	18.2	21.4
H10M			13.9		10.9		6.8				2.7	15.8	17.8	18.7	22.1
HSIP		13.5		10.6			6.6				2.7	15.4	17.3	18.3	21.6
HSIG		13.9			10.9		6.8				2.7	15.8	17.8	18.8	22.1
HSIM		14.2				11.2	6.8				2.8	16.2	18.3	19.3	22.7
HBAR		13.9			11.0		6.9				2.7	15.9	17.9	18.8	22.2
HMLE		13.8			10.9			6.8			2.7	15.8	17.7	18.7	22.1
HCDF		13.8			10.9				6.8		2.7	15.7	17.7	18.7	22.0
HSPT		15.2			12.0					7.5	3.0	17.3	19.5	20.6	24.3
H200														15.6	

		H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM	HCDF	2.436	2.478	2.557	1.960	2.012	2.066	1.265	2.877
NORM	HBAR	2.414	2.456	2.534	1.942	1.994	2.048	1.253	2.851
NORM	HMLE	2.434	2.476	2.555	1.958	2.011	2.065	1.264	2.875

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31	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	13.7				10.8		6.7				2.7	15.6	17.6	18.5	21.9
H10H		13.9			10.0		6.8				2.7	15.9	17.8	18.8	22.2
H10M			10.3		11.0		7.0				2.8	16.3	18.3	19.3	22.8
HSIP		14.3		11.2			7.0				2.8	16.3	18.3	19.3	22.8
HSIG		14.6			11.5		7.2				2.9	16.7	18.7	19.8	23.3
HSIM		15.0				11.8	7.2				2.9	17.1	19.2	20.3	23.9
HBAR		14.9			11.7		7.3				2.9	16.9	19.0	20.1	23.7
HMLE		14.7			11.6			7.2			2.9	16.8	18.9	19.9	23.5
HCDF		15.3			11.8				7.4		2.9	17.1	19.3	20.3	24.0
HSPT		16.2			12.8					8.0	3.2	18.5	20.8	21.9	25.9
H200														18.0	

		H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM	HCDF	2.324	2.360	2.423	1.906	1.949	1.996	1.238	3.046
NORM	HBAR	2.353	2.389	2.453	1.930	1.974	2.021	1.253	3.084
NORM	HMLE	2.372	2.408	2.472	1.945	1.989	2.037	1.263	3.108

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32	H12P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H12P	14.1				11.1		6.9				2.8	16.1	18.1	19.1	22.5
H10H		14.3			11.3		7.0				2.8	16.3	18.4	19.4	22.9
H10M			14.8		11.6		7.3				2.9	15.8	18.9	20.0	23.6
HSIP		14.8		11.6			7.3				2.9	15.8	18.9	20.0	23.5
HSIG		15.1			11.8		7.4				3.0	17.2	19.3	20.4	24.0
HSIM		15.4				12.1	7.4				3.0	17.5	19.7	20.8	24.6
HBAR		16.0			12.6		7.9				3.1	18.3	20.6	21.7	25.6
HMLE		15.6			12.3			7.7			3.1	17.8	20.0	21.1	24.9
HCDF		16.2			12.7				8.0		3.2	18.5	20.8	21.9	25.9
HSPT		16.4			12.9					8.1	3.2	18.7	21.0	22.2	26.2
H200														19.4	

		H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM	HCDF	2.215	2.253	2.324	1.824	1.861	1.902	1.241	3.056
NORM	HBAR	2.238	2.276	2.347	1.843	1.880	1.921	1.253	3.087
NORM	HMLE	2.302	2.340	2.414	1.895	1.933	1.976	1.289	3.175

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29	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	13.1				10.3		6.4				2.6	14.9	16.8	17.7	20.9
H10H		13.2			10.0		6.5				2.6	15.1	17.0	17.9	21.1
H10M			13.5		10.6		6.7				2.7	15.4	17.3	18.3	21.6
HSIP		14.0		11.0			6.9				2.7	15.9	17.9	18.9	22.3
HSIG		14.3			11.3		7.0				2.8	16.3	18.4	19.4	22.9
HSIM		14.7				11.6	7.0				2.9	16.8	18.9	19.9	23.5
HBAR		14.9			11.7		7.3				2.9	17.0	19.1	20.2	23.8
HMLE		14.5			11.0			7.1			2.9	16.6	18.6	19.6	23.2
HCDF		14.9			11.9				7.4		2.9	17.3	19.2	20.2	23.9
HSPT		15.6			12.3					7.7	3.1	17.8	20.0	21.1	24.9
H200														16.1	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.233	2.256	2.307	1.870	1.919	1.972	1.252	2.742
NORM HBAR	2.235	2.258	2.310	1.872	1.921	1.974	1.253	2.745
NORM HMLE	2.296	2.320	2.373	1.923	1.974	2.028	1.288	2.820

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30	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	12.6				9.0		6.2				2.5	14.3	16.1	17.2	20.0
H10H		12.8			10.1		6.3				2.5	14.6	16.4	17.3	20.5
H10M			13.3		10.5		6.6				2.6	15.2	17.1	18.0	21.3
HSIP		13.0		10.2			6.4				2.6	14.8	16.7	17.6	20.8
HSIG		13.3			10.0		6.5				2.6	15.1	17.0	18.0	21.2
HSIM		13.6				10.7	6.5				2.7	15.5	17.4	18.4	21.7
HBAR		13.6			10.7		6.7				2.7	15.6	17.5	18.4	21.8
HMLE		13.5			10.6			6.6			2.6	15.4	17.3	18.2	21.5
HCDF		13.8			10.9				6.8		2.7	15.8	17.7	18.7	22.1
HSPT		15.1			11.8					7.4	3.0	17.2	19.3	20.4	24.0
H200														15.9	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.3[illegible]	2.361	2.453	1.882	1.923	1.969	1.236	2.933
NORM HBAR	2.344	2.395	2.448	1.909	1.951	1.997	1.253	2.975
NORM HMLE	2.372	2.423	2.518	1.932	1.974	2.021	1.268	3.011

131 33	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	15.1				11.0		7.4				3.0	17.3	19.4	20.5	24.2
H10H		15.5			12.0		7.6				3.0	17.7	19.9	21.0	24.8
H10M			16.3		12.0		8.0				3.2	18.6	20.9	22.1	26.0
HSIP		15.1		11.9			7.4				3.0	17.3	19.4	20.5	24.2
HSIG		15.5			12.2		7.6				3.0	17.7	19.9	21.0	24.8
HSIM		16.0				12.5	7.6				3.1	18.2	20.4	21.6	25.5
HBAR		15.8			12.0		7.8				3.1	18.0	20.2	21.4	25.2
HMLE		15.7			12.3			7.7			3.1	17.9	20.1	21.2	25.1
HCDF		15.6			12.3				7.7		3.1	17.8	20.0	21.1	24.9
HSPT		17.0			13.0					8.4	3.3	19.4	21.8	23.0	27.2
H200														24.4	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.469	2.539	2.660	1.940	1.991	2.045	1.268	3.971
NORM HBAR	2.441	2.502	2.630	1.919	1.969	2.022	1.253	3.926
NORM HMLE	2.455	2.516	2.646	1.939	1.980	2.034	1.261	3.949

135 34	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	14.3				11.2		7.0				2.8	16.3	18.3	19.3	22.8
H10H		14.6			11.5		7.2				2.9	16.6	18.7	19.7	23.3
H10M			15.2		11.9		7.5				3.0	17.5	19.4	20.5	24.2
HSIP		14.5		11.4			7.2				2.9	16.6	18.6	19.7	23.2
HSIG		14.9			11.7		7.3				2.9	17.0	19.1	20.2	23.8
HSIM		15.3				12.0	7.3				3.0	17.5	19.6	20.7	24.4
HBAR		15.1			11.8		7.4				3.0	17.2	19.3	20.4	24.0
HMLE		15.0			11.8			7.4			2.9	17.1	19.2	20.3	24.0
HCDF		15.3			12.0				7.5		3.0	17.4	19.6	20.6	24.4
HSPT		16.3			12.8					8.0	3.2	18.6	20.9	22.1	26.1
H200														18.4	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.385	2.436	2.532	1.908	1.956	2.010	1.236	3.063
NORM HBAR	2.417	2.470	2.567	1.934	1.983	2.037	1.253	3.106
NORM HMLE	2.424	2.477	2.574	1.940	1.989	2.043	1.257	3.114

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35	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	14.9				11.7		7.3				2.9	17.0	19.1	20.1	23.7
H10H		15.2			11.9		7.5				3.0	17.3	19.5	20.5	24.2
H10M			15.8		12.0		7.8				3.1	18.0	20.3	21.4	25.3
HSIP		15.2		12.0			7.5				3.0	17.4	19.5	20.6	24.3
HSIG		15.6			12.3		7.7				3.1	17.9	20.0	21.1	24.9
HSIM		16.0				12.6	7.7				3.1	18.2	20.5	21.6	25.5
HBAR		15.6			12.3		7.7				3.1	17.8	20.0	21.1	24.9
HMLE		15.7			12.3			7.7			3.1	17.9	20.1	21.2	25.0
HCDF		15.9			12.5				7.8		3.1	18.1	20.3	21.5	25.3
HSPT		17.0			13.6					8.5	3.4	19.8	22.2	23.5	27.7
H200														26.2	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.388	2.439	2.542	1.921	1.969	2.020	1.231	4.199
NORM HBAR	2.432	2.482	2.588	1.956	2.005	2.057	1.253	4.275
NORM HMLE	2.417	2.468	2.573	1.945	1.993	2.045	1.246	4.250

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36	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	15.6				12.3		7.7				3.1	17.8	20.0	21.1	24.9
H10H		16.0			12.5		7.8				3.1	18.2	20.4	21.6	25.5
H10M			16.6		13.1		8.2				3.3	19.0	21.3	22.5	26.6
HSIP		16.1		12.6			7.9				3.2	18.3	20.6	21.7	25.6
HSIG		16.4			12.9		8.1				3.2	18.7	21.0	22.2	26.2
HSIM		16.8				13.2	8.1				3.3	19.1	21.5	22.7	26.8
HBAR		17.3			13.6		8.5				3.4	19.7	22.1	23.4	27.6
HMLE		16.9			13.3			8.3			3.3	19.3	21.7	22.9	27.0
HCDF		17.5			13.7				8.6		3.4	19.9	22.4	23.6	27.9
HSPT		18.1			14.2					8.9	3.5	20.6	23.2	24.5	28.9
H200														21.1	

	H10P	H10H	H10H	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.275	2.324	2.424	1.841	1.879	1.922	1.238	3.072
NORM HBAR	2.304	2.353	2.455	1.864	1.903	1.946	1.253	3.110
NORM HMLE	2.349	2.400	2.503	1.901	1.941	1.984	1.278	3.172

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37	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	17.6				13.9		8.6				3.4	20.0	22.5	23.8	28.0
H10H		18.3			14.1		8.9				3.5	20.5	23.1	24.3	28.7
H10M			18.8		14.7		9.2				3.7	21.4	24.0	25.4	29.9
HSIP		17.5		13.8			8.6				3.4	20.0	22.5	23.7	28.0
HSIG		18.1			14.2		8.9				3.5	20.6	23.1	24.4	28.8
HSIM		18.6				14.6	8.9				3.6	21.1	23.8	25.1	29.6
HBAR		17.7			13.9		8.7				3.5	20.2	22.7	24.0	28.3
HMLE		17.9			14.0			8.8			3.5	20.3	22.9	24.1	28.5
HCDF		17.5			13.7				8.6		3.4	19.9	22.4	23.6	27.9
HSPT		18.8			14.8					9.2	3.7	21.4	24.1	25.4	30.0
H200														22.4	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.561	2.622	2.733	2.008	2.068	2.126	1.271	3.260
NORM HBAR	2.526	2.586	2.696	1.980	2.040	2.097	1.253	3.219
NORM HMLE	2.509	2.569	2.678	1.967	2.027	2.083	1.245	3.198

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38	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	18.1				14.2		8.9				3.6	20.7	23.2	24.5	28.9
H10H		18.5			14.6		9.1				3.6	21.1	23.8	25.1	29.6
H10M			19.4		15.3		9.6				3.8	22.2	24.9	26.3	31.0
HSIP		17.7		13.9			8.7				3.5	20.2	22.7	24.0	28.3
HSIG		18.3			14.4		9.0				3.6	20.9	23.5	24.7	29.2
HSIM		18.8				14.8	9.0				3.7	21.5	24.1	25.5	30.1
HBAR		17.8			14.0		8.7				3.5	20.2	22.8	24.0	28.3
HMLE		18.0			14.2			8.9			3.5	20.6	23.1	24.4	28.8
HCDF		17.4			13.7				8.6		3.4	19.9	22.4	23.6	27.9
HSPT		20.0			15.7					9.8	3.9	22.8	25.6	27.1	31.9
H200														25.7	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.644	2.707	2.838	2.035	2.100	2.161	1.275	3.750
NORM HBAR	2.599	2.661	2.790	2.001	2.064	2.124	1.253	3.686
NORM HMLE	2.559	2.620	2.746	1.970	2.032	2.091	1.234	3.629

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49	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	18.9				14.2		8.9				3.5	20.5	23.1	24.3	28.7
H10H		18.4			14.5		9.1				3.6	21.0	23.6	24.9	29.4
H10M			19.1		15.1		9.4				3.8	21.8	24.5	25.9	30.6
HSIP		18.2		14.3			9.0				3.6	20.7	23.3	24.6	29.0
HSIG		18.7			14.7		9.2				3.7	21.3	23.9	25.2	29.8
HSIM		19.2				15.1	9.2				3.8	21.9	24.6	26.0	30.6
HBAR		19.2			15.1		9.4				3.8	21.9	24.6	25.9	30.6
HMLE		19.0			14.9			9.3			3.7	21.6	24.3	25.7	30.3
HCDF		19.4			15.3				9.6		3.8	22.1	24.9	26.2	31.0
HSPT		22.9			16.4					10.3	4.1	23.8	26.8	28.2	33.3
H200														23.5	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.363	2.414	2.513	1.877	1.926	1.989	1.239	3.081
NORM HBAR	2.390	2.443	2.542	1.899	1.949	2.003	1.253	3.117
NORM HMLE	2.415	2.468	2.569	1.918	1.969	2.024	1.266	3.149

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40	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	18.3				14.4		9.2				3.6	20.9	23.4	24.7	29.2
H10H		18.6			14.7		9.2				3.7	21.3	23.9	25.2	29.8
H10M			19.4		15.2		9.5				3.8	22.1	24.8	26.2	30.9
HSIP		18.5		14.6			9.1				3.6	21.1	23.7	25.1	29.6
HSIG		19.0			14.9		9.4				3.7	21.7	24.4	25.7	30.3
HSIM		19.5				15.4	9.4				3.8	22.3	25.0	26.4	31.2
HBAR		19.0			15.0		9.4				3.7	21.7	24.4	25.8	30.4
HMLE		19.2			15.1			9.4			3.8	21.8	24.6	25.9	30.6
HCDF		19.5			15.3				9.6		3.8	22.2	24.9	26.3	31.1
HSPT		21.0			16.5					10.3	4.1	24.0	27.0	28.4	33.6
H200														26.0	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.390	2.440	2.536	1.907	1.956	2.011	1.227	3.406
NORM HBAR	2.446	2.492	2.591	1.947	1.998	2.054	1.253	3.480
NORM HMLE	2.432	2.478	2.575	1.936	1.986	2.042	1.246	3.459

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41	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	18.9				14.8		9.3				3.7	21.5	24.2	25.5	30.1
H10H		19.1			15.0		9.4				3.8	21.8	24.5	25.9	30.5
H10M			19.6		15.0		9.6				3.8	22.3	25.1	26.5	31.2
HSIP		20.1		15.8			9.9				3.9	22.9	25.7	27.1	32.0
HSIG		20.5			16.2		10.1				4.0	23.4	26.3	27.8	32.8
HSIM		21.1				16.6	10.1				4.1	24.0	27.0	28.5	33.6
HBAR		21.1			16.6		10.4				4.1	24.1	27.1	28.6	33.7
HMLE		20.8			16.4			10.3			4.1	23.7	26.7	28.2	33.2
HCDF		21.7			17.0				10.7		4.3	24.7	27.8	29.3	34.6
HSPT		22.4			17.6					11.0	4.4	25.6	28.8	30.3	35.8
H200														26.1	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.220	2.250	2.300	1.855	1.899	1.947	1.221	3.071
NORM HBAR	2.278	2.309	2.360	1.904	1.949	1.998	1.253	3.152
NORM HMLE	2.309	2.340	2.392	1.930	1.975	2.020	1.270	3.194

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42	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	21.2				16.6		10.4				4.2	24.1	27.1	28.6	33.8
H10H		21.5			16.0		10.6				4.2	24.5	27.5	29.0	34.3
H10M			22.0		17.3		10.8				4.3	25.1	28.2	29.8	35.1
HSIP		22.4		17.6			11.0				4.4	25.5	28.7	30.3	35.8
HSIG		23.0			18.1		11.3				4.5	26.2	29.4	31.0	36.6
HSIM		23.5				18.5	11.3				4.6	26.8	30.1	31.8	37.5
HBAR		23.8			18.7		11.7				4.7	27.2	30.5	32.2	38.0
HMLE		23.3			18.0			11.5			4.6	26.6	29.9	31.6	37.3
HCDF		24.2			19.1				11.9		4.8	27.6	31.1	32.8	38.7
HSPT		24.5			19.3					12.1	4.8	28.0	31.5	33.2	39.2
H200														25.6	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.223	2.256	2.313	1.851	1.896	1.941	1.231	2.685
NORM HBAR	2.264	2.297	2.355	1.888	1.931	1.977	1.253	2.734
NORM HMLE	2.309	2.342	2.402	1.922	1.969	2.016	1.278	2.788

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43	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	22.5				17.7		11.1				4.4	25.6	28.8	30.4	35.9
H10H		22.2			18.0		11.3				4.5	26.1	29.4	31.2	36.6
H10M			23.8		18.7		11.7				4.7	27.1	30.5	32.2	38.0
HSIP		22.9		18.0			11.2				4.5	26.1	29.3	30.9	36.5
HSIG		23.4			18.6		11.5				4.6	26.7	30.0	31.7	37.4
HSIM		24.0				18.9	11.5				4.7	27.4	30.8	32.5	38.4
HBAR		23.5			18.5		11.6				4.6	26.8	30.1	31.8	37.5
HMLE		23.4			18.4			11.5			4.6	26.7	30.0	31.6	37.3
HCDF		23.3			18.3				11.5		4.6	26.6	29.9	31.5	37.2
HSPT		25.2			19.8					12.4	5.0	28.8	32.4	34.1	40.3
H200														29.1	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.460	2.508	2.603	1.965	2.015	2.068	1.264	3.178
NORM HBAR	2.439	2.486	2.581	1.948	1.997	2.050	1.253	3.150
NORM HMLE	2.448	2.496	2.591	1.956	2.005	2.058	1.258	3.163

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44	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	25.1				19.9		12.4				4.9	28.7	32.2	34.0	40.1
H10H		25.7			20.2		12.6				5.0	29.3	32.9	34.7	41.0
H10M			26.8		21.1		13.2				5.3	30.6	34.4	36.3	42.8
HSIP		26.1		20.5			12.8				5.1	29.7	33.4	35.3	41.6
HSIG		26.7			21.0		13.1				5.2	30.5	34.2	36.1	42.6
HSIM		27.0				21.5	13.1				5.4	31.2	35.1	37.2	43.7
HBAR		26.9			21.1		13.2				5.3	30.6	34.4	36.3	42.9
HMLE		26.8			21.1			13.2			5.3	30.6	34.4	36.3	42.8
HCDF		27.2			21.0				13.4		5.3	31.1	34.9	36.9	43.5
HSPT		29.3			23.0					14.4	5.8	33.4	37.5	39.6	46.8
H200														37.1	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.350	2.399	2.506	1.917	1.960	2.013	1.236	3.467
NORM HBAR	2.383	2.432	2.541	1.944	1.991	2.002	1.253	3.515
NORM HMLE	2.387	2.436	2.545	1.947	1.994	2.045	1.255	3.521

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45	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	26.5				20.0		13.1				5.2	30.3	34.0	35.9	42.4
H10H		27.1			21.3		13.3				5.3	30.9	34.7	36.6	43.2
H10M			28.1		22.1		13.8				5.5	32.1	36.1	38.1	44.9
HSIP		26.0		21.2			13.2				5.3	30.7	34.5	36.4	43.0
HSIG		27.5			21.7		13.6				5.4	31.4	35.3	37.3	44.0
HSIM		28.2				22.2	13.6				5.5	32.2	36.2	38.2	45.1
HBAR		28.2			22.1		13.9				5.5	32.1	36.1	38.1	44.9
HMLE		27.9			22.0			13.7			5.5	31.8	35.8	37.8	44.6
HCDF		28.4			22.8				14.0		5.6	32.0	36.0	38.5	45.4
HSPT		29.8			23.5					14.7	5.9	34.0	38.2	40.3	47.6
H200														33.6	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.377	2.425	2.520	1.895	1.940	1.989	1.241	3.011
NORM HBAR	2.391	2.449	2.545	1.914	1.959	2.008	1.253	3.041
NORM HMLE	2.421	2.469	2.566	1.930	1.975	2.025	1.264	3.066

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46	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	30.0				23.6		14.8				5.9	34.2	38.4	40.6	47.9
H10H		30.5			24.0		15.0				6.0	34.8	39.1	41.3	48.7
H10M			31.6		24.8		15.5				6.2	36.0	40.5	42.7	50.4
HSIP		30.8		24.2			15.1				6.0	35.1	39.4	41.6	49.1
HSIG		31.5			24.8		15.5				6.2	35.9	40.4	42.6	50.3
HSIM		32.3				25.4	15.5				6.3	36.8	41.4	43.7	51.6
HBAR		31.8			25.0		15.6				6.2	36.2	40.7	43.0	50.7
HMLE		31.7			24.9			15.5			6.2	36.1	40.6	42.8	50.6
HCDF		32.4			25.5				15.9		6.4	36.9	41.5	43.8	51.7
HSPT		33.2			26.1					16.3	6.5	37.8	42.5	44.9	53.8
H200														36.8	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.358	2.399	2.485	1.901	1.949	1.997	1.229	2.890
NORM HBAR	2.405	2.447	2.533	1.939	1.987	2.037	1.253	2.947
NORM HMLE	2.411	2.453	2.540	1.944	1.993	2.042	1.257	2.955

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47	H10P	H10H	H10M	HSIP	HSIG	HSIM	HHAR	HMLE	HCDF	HSPT	F0	H 5	HMP	H200	H 95
H10P	33.9				26.6		16.6				6.6	38.6	43.4	45.8	54.0
H10H		34.3			27.0		16.9				6.7	39.1	44.0	46.4	54.8
H10M			35.1		27.6		17.3				6.9	40.2	44.9	47.4	56.0
HSIP		34.7		27.3			17.1				6.8	39.6	44.5	46.9	55.4
HSIG		35.6			28.0		17.5				7.0	40.6	45.6	48.1	56.8
HSIM		36.6				28.8	17.5				7.2	41.7	46.9	49.5	58.4
HHAR		35.6			28.0		17.5				7.2	40.5	45.6	48.1	56.8
HMLE		35.5			27.9			17.5			7.0	40.5	45.5	48.0	56.7
HCDF		35.8			28.2				17.6		7.0	40.8	45.9	48.4	57.2
HSPT		37.9			29.8					18.7	7.4	43.2	48.6	51.3	60.5
H200														37.2	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HHAR	H200
NORM HCDF	2.407	2.449	2.494	1.941	1.999	2.045	1.245	2.646
NORM HHAR	2.424	2.458	2.512	1.955	2.010	2.060	1.253	2.665
NORM HMLE	2.428	2.462	2.516	1.959	2.008	2.063	1.255	2.669

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48	H10P	H10H	H10M	HSIP	HSIG	HSIM	HHAR	HMLE	HCDF	HSPT	F0	H 5	HMP	H200	H 95
H10P	37.4				29.4		18.4				7.3	42.6	47.9	50.5	59.6
H10H		38.4			30.2		18.9				7.5	43.9	49.2	51.9	61.3
H10M			40.7		32.0		20.0				8.0	46.4	52.2	55.1	65.0
HSIP		36.7		28.9			18.1				7.2	41.8	47.0	49.6	58.6
HSIG		37.7			29.6		18.5				7.4	43.0	48.3	51.0	60.2
HSIM		38.8				30.5	18.5				7.6	44.2	49.7	52.4	61.9
HHAR		38.3			30.1		18.8				7.5	43.6	49.0	51.7	61.1
HMLE		38.4			30.2			19.9			7.5	43.8	49.2	51.9	61.3
HCDF		37.8			29.7				18.6		7.4	43.1	48.5	51.2	60.4
HSPT		40.8			32.1					20.1	8.0	46.5	52.3	55.2	65.1
H200														72.2	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HHAR	H200
NORM HCDF	2.516	2.585	2.743	1.943	1.996	2.053	1.268	4.860
NORM HHAR	2.487	2.555	2.712	1.921	1.973	2.030	1.253	4.808
NORM HMLE	2.479	2.547	2.703	1.915	1.967	2.023	1.249	4.793

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49	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	37.0				29.1		18.2				7.3	42.2	47.4	50.0	59.1
H10H		37.8			29.7		18.6				7.4	43.1	48.5	51.2	60.4
H10M			39.3		30.9		19.3				7.7	44.8	50.3	53.1	62.7
HSIP		37.0		29.1			18.2				7.3	42.2	47.5	50.1	59.1
HSIG		38.0			29.0		18.7				7.5	43.3	48.7	51.4	60.7
HSIM		39.2				30.8	18.7				7.7	44.6	50.2	53.0	62.5
HBAR		38.2			30.0		18.8				7.5	43.5	48.9	51.6	60.9
HMLE		38.3			30.1			18.8			7.5	43.6	49.0	51.7	61.1
HCDF		38.5			30.3				19.0		7.6	43.9	49.4	52.1	61.5
HSPT		41.0			32.3					22.2	8.1	46.8	52.6	55.5	65.5
H200														46.5	

		H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM	HCDF	2.445	2.500	2.595	1.925	1.975	2.035	1.241	3.070
NORM	HBAR	2.472	2.525	2.621	1.944	1.995	2.055	1.253	3.101
NORM	HMLE	2.463	2.519	2.614	1.939	1.990	2.050	1.250	3.093

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50	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	38.9				30.6		19.1				7.6	44.4	49.9	52.6	62.1
H10H		39.9			31.0		19.6				7.9	45.5	51.1	53.9	63.7
H10M			41.9		33.0		20.6				8.2	47.8	53.7	56.7	66.9
HSIP		38.9		30.6			19.2				7.6	44.4	49.9	52.7	62.2
HSIG		40.0			31.0		19.7				7.8	45.6	51.2	54.0	63.8
HSIM		41.1				32.3	19.7				8.1	46.9	52.7	55.6	65.6
HBAR		39.6			31.1		19.5				7.8	45.2	50.8	53.6	63.2
HMLE		40.1			31.6			19.7			7.9	45.8	51.4	54.3	64.1
HCDF		40.1			31.5				19.7		7.9	45.7	51.4	54.2	64.0
HSPT		42.0			33.0					20.7	8.2	47.9	53.8	56.8	67.1
H200														65.4	

		H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM	HCDF	2.472	2.534	2.663	1.945	1.996	2.053	1.238	4.155
NORM	HBAR	2.502	2.565	2.695	1.969	2.021	2.078	1.253	4.206
NORM	HMLE	2.469	2.531	2.660	1.943	1.994	2.051	1.237	4.151

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S1	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	44.7				35.1		22.0				8.8	51.9	57.3	60.4	71.3
H10H		45.7			35.0		22.5				9.0	52.1	58.5	61.8	72.9
H10M			47.6		37.0		23.4				9.3	54.2	60.9	64.3	75.9
HSIP		44.5		35.0			21.9				8.7	50.8	57.1	60.2	71.1
HSIG		45.7			36.0		22.5				9.0	52.2	58.6	61.9	73.0
HSIM		47.1				37.1	22.5				9.3	53.7	60.0	63.7	75.2
HBAR		46.6			36.6		22.9				9.1	53.1	59.7	63.2	74.3
HMLE		46.0			36.0			22.8			9.1	52.9	59.4	62.7	74.0
HCDF		46.8			36.8				23.0		9.2	53.3	59.9	63.3	74.7
HSPT		46.3			36.0					22.8	9.1	52.8	59.3	62.6	73.9
H200														58.9	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.434	2.488	2.590	1.906	1.959	2.018	1.248	3.210
NORM HBAR	2.446	2.499	2.602	1.915	1.969	2.028	1.253	3.225
NORM HMLE	2.456	2.510	2.613	1.923	1.976	2.036	1.259	3.238

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S2	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	38.5				30.3		19.0				7.6	43.9	49.4	52.1	61.5
H10H		39.3			30.0		19.3				7.7	44.8	50.3	53.1	62.7
H10M			40.8		32.1		20.1				8.0	46.6	52.3	55.2	65.2
HSIP		39.6		31.1			19.5				7.8	45.1	50.7	53.5	63.2
HSIG		40.5			31.0		19.9				8.0	46.2	51.9	54.8	64.7
HSIM		41.5				32.7	19.9				8.2	47.4	53.2	56.2	66.3
HBAR		41.3			32.0		20.3				8.1	47.0	52.9	55.8	65.8
HMLE		41.1			32.3			20.2			8.1	46.9	52.7	55.6	65.6
HCDF		42.0			33.3				20.9		8.3	48.3	54.3	57.3	67.7
HSPT		44.5			35.0					21.9	8.7	50.8	57.1	60.2	71.1
H200														45.8	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.316	2.361	2.454	1.870	1.914	1.963	1.220	2.750
NORM HBAR	2.380	2.425	2.522	1.921	1.967	2.017	1.253	2.826
NORM HMLE	2.388	2.433	2.530	1.928	1.973	2.024	1.258	2.835

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53	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	42.2				33.2		20.8				8.3	48.2	54.1	57.1	67.0
H10H		42.8			33.7		21.1				8.0	48.8	54.9	57.9	68.3
H10M			44.0		34.6		21.6				8.6	50.1	56.4	59.5	70.2
HSIP		44.6		35.0			21.9				8.8	50.8	57.1	60.3	71.2
HSIG		45.6			35.8		22.4				9.0	52.0	58.4	61.7	72.8
HSIM		46.7				36.7	22.4				9.2	53.2	59.8	63.1	74.5
HBAR		45.9			36.0		22.5				9.0	52.2	58.7	61.9	73.1
HMLE		45.6			35.8			22.4			8.9	51.9	58.4	61.6	72.7
HCDF		46.6			36.6				22.9		9.1	53.1	59.7	63.0	74.4
HSPT		47.5			37.0					23.4	9.3	54.2	60.9	64.3	75.9
H200														50.7	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.309	2.341	2.404	1.916	1.960	2.007	1.232	2.769
NORM HBAR	2.350	2.383	2.446	1.953	1.994	2.043	1.253	2.818
NORM HMLE	2.362	2.394	2.458	1.960	2.004	2.053	1.260	2.832

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50	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	46.5				36.5		22.9				9.1	53.0	59.6	62.8	74.2
H10H		47.0			37.3		23.3				9.3	54.0	60.7	64.1	75.6
H10M			49.4		38.0		24.3				9.7	56.3	63.3	66.7	78.8
HSIP		47.7		37.5			23.5				9.4	54.4	61.2	64.6	76.2
HSIG		49.0			38.5		24.1				9.6	55.9	62.8	66.3	78.2
HSIM		50.3				39.5	24.1				9.9	57.3	64.4	68.0	80.3
HBAR		50.1			39.0		24.6				9.8	57.1	64.2	67.7	79.9
HMLE		49.4			38.8			24.3			9.7	56.3	63.3	66.8	78.9
HCDF		49.6			39.0				24.4		9.7	56.5	63.6	67.1	79.1
HSPT		50.7			39.9					25.0	10.0	57.8	65.0	68.6	80.9
H200														68.5	

	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM HCDF	2.387	2.435	2.536	1.928	1.980	2.031	1.266	3.517
NORM HBAR	2.364	2.411	2.511	1.910	1.961	2.012	1.253	3.483
NORM HMLE	2.396	2.443	2.545	1.935	1.987	2.039	1.270	3.530

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SS	H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	HMLE	HCDF	HSPT	E0	H 5	HMP	H200	H 95
H10P	52.2				41.0		25.7				10.2	59.5	66.9	70.5	83.3
H10H		53.2			41.0		26.2				10.4	60.7	68.2	72.0	85.0
H10M			55.6		43.7		27.4				10.9	63.4	71.2	75.2	88.7
HSIP		52.7		41.5			26.0				10.0	60.1	67.6	71.3	84.2
HSIG		54.1			42.6		26.6				10.6	61.7	69.4	73.2	86.4
HSIM		55.7				43.8	26.6				10.9	63.5	71.3	75.3	88.8
HBAR		53.2			41.8		26.2				10.4	60.7	68.2	71.9	84.9
HMLE		53.8			42.3			26.5			10.6	61.5	68.9	72.8	85.9
HCDF		53.7			42.2				26.4		10.5	61.2	68.8	72.5	85.6
HSPT		56.2			44.2					27.7	11.3	64.1	72.1	76.1	89.9
H200														70.5	

		H10P	H10H	H10M	HSIP	HSIG	HSIM	HBAR	H200
NORM	HCDF	2.477	2.527	2.639	1.968	2.021	2.377	1.243	3.340
NORM	HBAR	2.498	2.549	2.661	1.985	2.038	2.395	1.253	3.368
NORM	HMLE	2.470	2.520	2.631	1.963	2.015	2.372	1.239	3.330

END

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